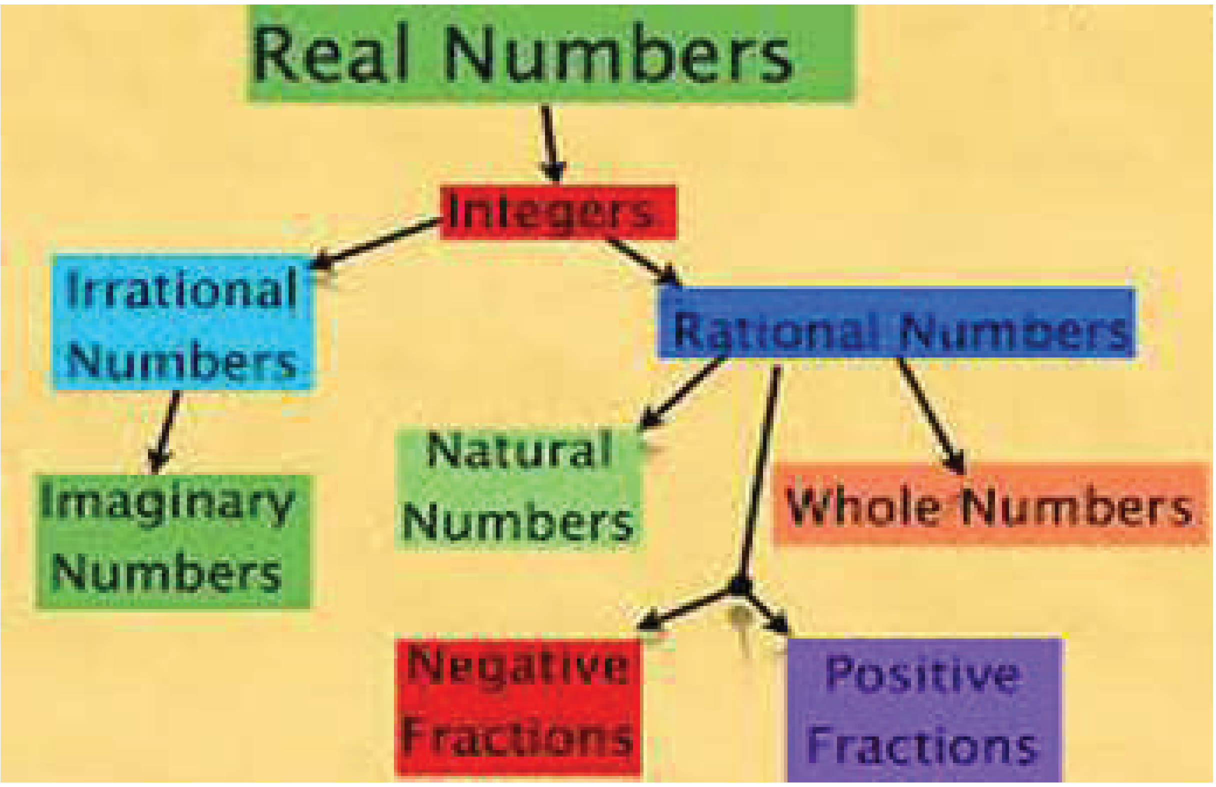
CHAPTER –1. NUMBER SYSTEM



KEY POINTS-

Introduction to Natural Numbers
Non-negative counting numbers excluding zero are called Natural Numbers.

N = {1, 2, 3, 4,5…}

Whole Numbers
All natural numbers including zero are called Whole Numbers.

W = {0, 1, 2, 3, 4, 5…}

Integers
All natural numbers, 0 and negatives of natural numbers are called Integers.

Z = {……,–3,–2,–1, 0, 1, 2, 3, 4…….}

Rational Numbers
The number 'a' is called Rational if it can be written in the form of r/s where 'r' and 's'

are integers and s ≠ 0,

Q = 2/3, 3/5,7 etc. all are rational numbers.

How to find a rational number between two given numbers?

The rational number between two given numbers ‘a’ and ‘b’ is

$$\frac{a+b}{2}$$

Example: Find two rational numbers between 4 and 5.

Solution: To find a rational number between 4 and 5.

$$\frac{a+b}{2} = \frac{4+5}{2} = \frac{9}{2}$$

To find another number we will follow the same process again.

$$\frac{1}{2}\left(4+\frac{9}{2}\right) = \left(\frac{1}{2}\right)\frac{17}{2} = \frac{17}{4}$$

Hence the two rational numbers between 4 and 5 are 9/2 and 17/4.

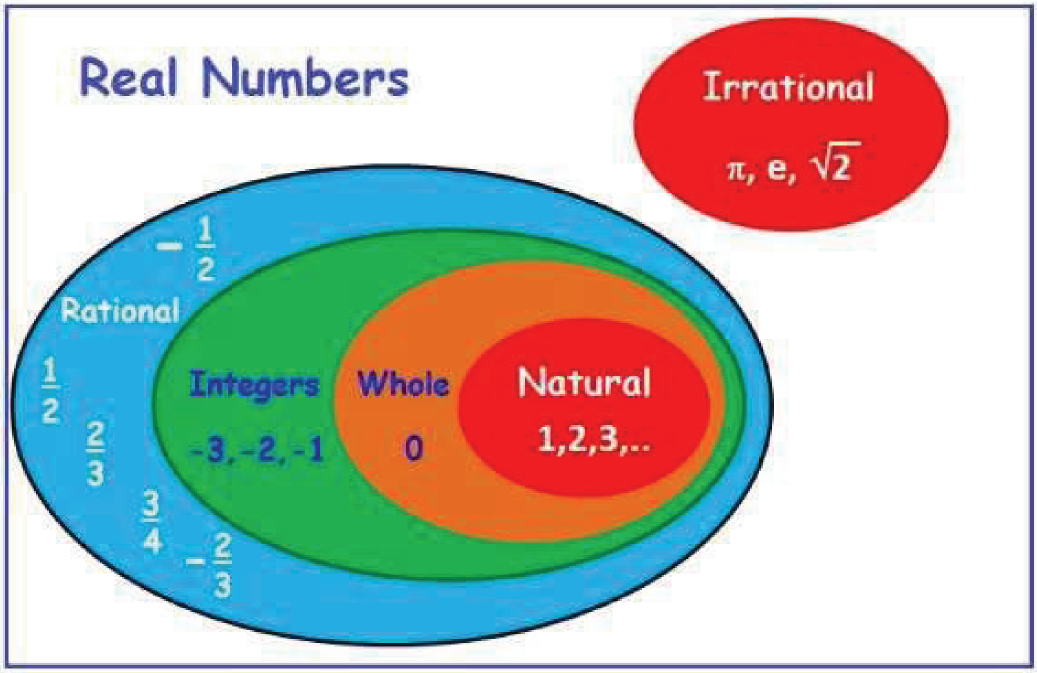
Remarks: There could be unlimited rational numbers between any two rational numbers.

Irrational Numbers

The number ‘a’ which cannot be written in the form of p/q is called irrational, where p and q are integers and q ≠ 0 or you can say that the numbers which are not rational are called Irrational Numbers.**Example -** √7, √11 etc.

Real Numbers

All numbers including both rational and irrational numbers are called Real Numbers R = –2, (-2/3), 0, 3 and √2 etc.



Real Numbers and their Decimal Expansions

1. Rational Number

If the rational number is in the form of a/b then by dividing a by b we can get two situations.

a. If the remainder becomes zero

While dividing if we get zero as the remainder after some steps then the decimal expansion of such number is called terminating.

Example: 7/8 = 0.875

b. If the remainder does not become zero

While dividing if the decimal expansion continues and a digit or a set of finite number of digits repeats periodically then it is called non terminating recurring or repeating decimal expansion.

Example: 1/3 = 0.3333….

It can be written as $0.\overline{3}$

Hence, the decimal expansion of rational numbers could be terminating or non-terminating recurring and vice-versa.

2. Irrational Numbers

If we do the decimal expansion of an irrational number then it would be non – terminating non- recurring and vice-versa. i.e the remainder does not become zero and also not repeated.

Example: $\pi = 3.141592653589793238\ldots\ldots$

Operations on Real Numbers

1. The sum, difference, product and quotient of two rational numbers will be

$$\Rightarrow \frac{3}{4} + \frac{7}{4} = \frac{10}{4} = \frac{5}{2}$$

$$\Rightarrow \frac{7}{4} - \frac{3}{4} = \frac{4}{4} = 1$$

$$\Rightarrow \frac{7}{4} \times \frac{3}{4} = \frac{21}{16}$$

$$\Rightarrow \frac{7}{4} \div \frac{3}{4} = \frac{7}{3}$$

rational.**Example:**

2. If we add or subtract a rational number with an irrational number then the outcome will be irrational.

Example: If 5 is a rational number and $\sqrt{7}$ is an irrational number then $5+\sqrt{7}$ and $5-\sqrt{7}$ are irrational numbers.

3. If we multiply or divide a non-zero rational number with an irrational number then also the outcome will be irrational.

Example: If 7 is a rational number and $\sqrt{5}$ is an irrational number then $7\sqrt{7}$ and $7/\sqrt{5}$ are irrational numbers.

4. The sum, difference, product and quotient of two irrational numbers could be rational or irrational.

$$\sqrt{3} + \sqrt{3} = 2\sqrt{3} \qquad \text{(irrational + iratinal = irrational)}$$

$$\sqrt{2} - \sqrt{2} = 0 \qquad \text{(irrational – irrational = rational)}$$

$$(\sqrt{6}).(\sqrt{6}) = 6 \qquad \text{(irrational × irrational = rational)}$$

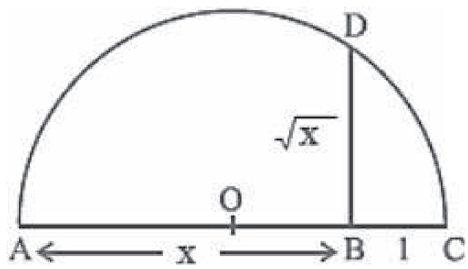
$$\frac{\sqrt{13}}{\sqrt{13}} = 1 \qquad \text{(irrational ÷ irrational = rational)}$$

Example:

Finding Roots of a Positive Real Number 'x' geometrically and mark it on the Number Line

To find $\sqrt{x}$ geometrically

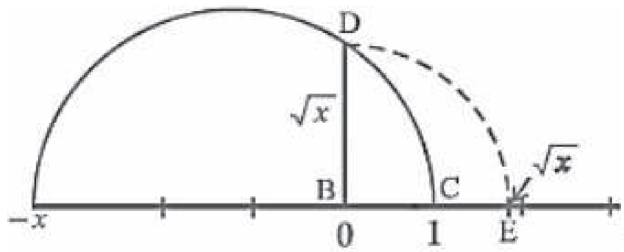
1. First of all, mark the distance x unit from point A on the lines B that AB = x unit.
2. From B mark a point C with the distance of 1unit, so that BC = 1unit.
3. Take the midpoint of AC and mark it as O. Then take OC as the radius and draw a semi circle.
4. From the point B draw a perpendicular BD which intersects the semi circle at point D.



The length of BD = $\sqrt{x}$ unit.

To mark the position of $\sqrt{x}$ on the number line, we will take AC as the number line, with B as zero. So C is point 1 on the number line.

Now we will take B as the centre and BD as the radius, and draw an arc intersecting the number line at point E.



Now E is $\sqrt{x}$ on the number line.

Identities Related to Square Roots

If p and q are two positive real numbers

1. $\sqrt{pq} = \sqrt{p}\sqrt{q}$
2. $\sqrt{\frac{p}{q}} = \frac{\sqrt{p}}{\sqrt{q}}$
3. $(\sqrt{p} + \sqrt{q})(\sqrt{p} - \sqrt{q}) = p - q$
4. $(p + \sqrt{q})(p - \sqrt{q}) = p^2 - q$
5. $(\sqrt{p} + \sqrt{q})(\sqrt{r} + \sqrt{s}) = \sqrt{pr} + \sqrt{ps} + \sqrt{qr} + \sqrt{qs}$
6. $(\sqrt{p} + \sqrt{q})^2 = p + 2\sqrt{pq} + q$

Examples: Simplify

$$(\sqrt{5} + \sqrt{11})(\sqrt{5} - \sqrt{11})$$

We will use the identity

$$(\sqrt{p} + \sqrt{q})(\sqrt{p} - \sqrt{q}) = p - q$$
$$(\sqrt{5} + \sqrt{11})(\sqrt{5} - \sqrt{11}) = 5 - 11 = -6$$

Rationalizing the Denominator

Rationalize the denominator means to convert the denominator containing square root term into a rational number by finding the equivalent fraction of the given fraction.

For which we can use the identities of the real numbers.

To rationalise the denominator of $\frac{1}{\sqrt{a} + b}$, we multiply this by $\frac{\sqrt{a} - b}{\sqrt{a} - b}$, where a and b are integers.

Example: Rationalize the denominator of 7 / (7-√3).

Solution: We will use the identity $(p + \sqrt{q})(p - \sqrt{q}) = p^2 - q$ here.

$$\frac{7}{7 - \sqrt{3}} \times \frac{7 + \sqrt{3}}{7 + \sqrt{3}} = \frac{7(7 + \sqrt{3})}{49 - 3} = \frac{49 + 7\sqrt{3}}{46}$$

Laws of Exponents for Real Numbers

If we have a and b as the base and m and n as the exponents, then

1. $a^m \times a^n = a^{m+n}$

2. $(a^m)^n = a^{mn}$

3. $\frac{a^m}{a^n} = a^{m-n}, m > n$

4. $a^m b^m = (a b)^m$

5. $a^0 = 1$

6. $a^1 = a$

7. $1/a^n = a^{-n}$

- Let a > 0 be a real number and n a positive integer.

$$\text{Then } \sqrt[n]{a} = b, \text{if } b^n = a \text{ and } b > 0$$

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

- Let a > 0 be a real number. Let m and n be integers such that m and n have no common factors other than 1, and n > 0. Then,

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$