Unit I: Electrostatics

CHAPTER–2: ELECTROSTATIC POTENTIAL AND CAPACITANCE

GIST OF THE CHAPTER:-

Electric potential, potential difference, electric potential due to a point charge, a dipole and system of charges; equipotential surfaces, electrical potential energy of a system of two-point charges and of electric dipole in an electrostatic field.

Conductors and insulators, free charges and bound charges inside a conductor. Dielectrics and electric polarization, capacitors and capacitance, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, energy stored in a capacitor (no derivation, formulae only).

DEFINITIONS & CONCEPTS:-

1. **Electric potential**. The electric potential at a point in an electric field is defined as the amount of work done per unit positive test charge in moving the test charge from infinity to that point against the electrostatic force due to the field.

2. Electric potential difference. The electric potential difference between two points in an electric field is defined as theamount of work done per unit positive test charge in moving the test charge from one point to the other against the electrostatic force due to the field of charge Q. Its unit is volt

Mathematically: If W is work done in moving a small positive test charge q, from point A to B in the electrostatic field of charge Q, then potential difference between points B and A,

$$V_A - V_B = \frac{W_{AB}}{q} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

3. **Electric potential due to group of charges.** The electric potential at a point due to a group of charges is equal to the algebraic sum of the electric potentials due to individual charges at that point. It is a scalar quantity.

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + - - - - - - - + \frac{q_n}{r_n}\right)$$

4. **Potential gradient.** The rate of change of potential with distance at a point is called potential gradient at that point. The electric field at a point is equal to the negative potential gradient at that point. E = - dv/dr.

5. **Electric potential at a point due to a dipole**:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{\boldsymbol{p}}.\hat{\boldsymbol{r}}}{r^2}$$

Equipotential Surfaces:

A surface with a constant value of potential at all points on the surface. Example: Surface of a charged conductor.

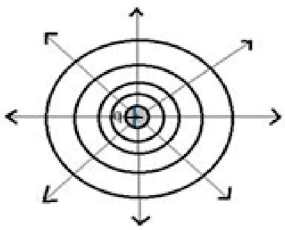
Properties of Equipotential Surfaces:

(i) No work is required to move a test charge on the equipotential surface.
(ii) The electric field is always normal to the equipotential surface at every point.
(iii) No two equipotential surfaces can intersect each other.
(iv) These are closer in the regions of strong electric fields and farther apart in the regions of weak field.

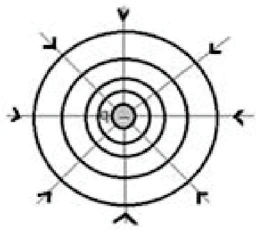
Equipotential Surfaces for various charge systems

For isolated point charge - concentric spheres

(Positive charge)

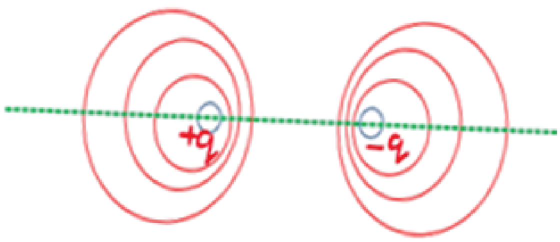


(Negative charge)

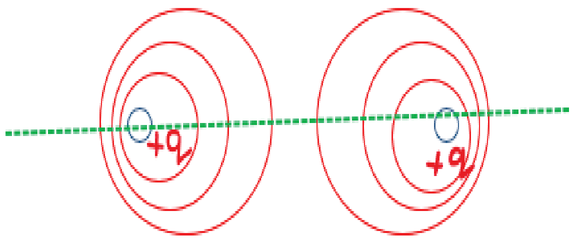


For electric dipole

(system of two equal and opposite charges)

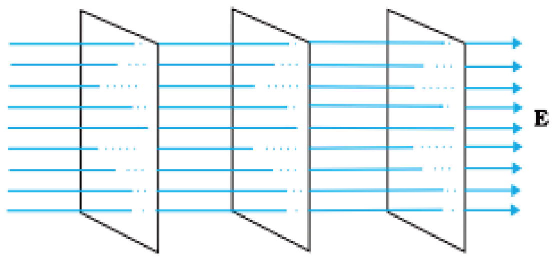


For like charges:

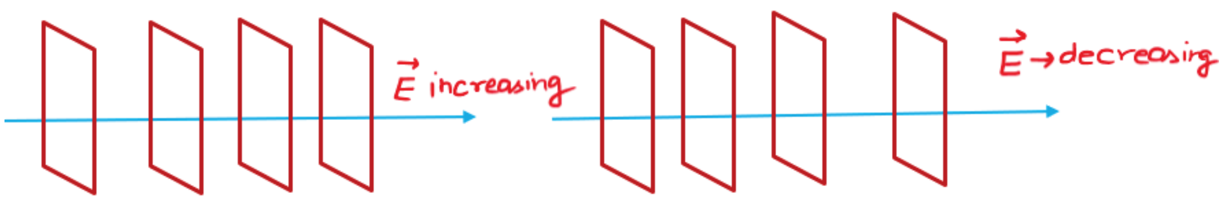


For uniform electric field:

Parallel planes perpendicular to the electric field



For non-uniform electric field:



Electric Potential Energy

Electric P.E. (U) is an amount of work done in assembling the charges at their locations by bringing them in, from infinity.

Note that **U is +ve** for like charges and **-ve** for unlike charges.

For Potential energy of a single point charge:

$$U(\mathbf{r}) = q\,V(\mathbf{r})$$

For Potential energy of a system of two-point charges:

$$U(\mathbf{r}) = q_1 V(\mathbf{r}_1) + q_2 V(\mathbf{r}_2) + \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}}$$

For Potential Energy of a system of three-point charges:

$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

Potential Energy in an External Field-

$$\mathbf{U = -pE\cos\theta = -\vec{p}.\vec{E}}$$

CONDUCTORS, INSULATORS:-

On the basis of their behaviour in an external field, material can be classified into two categories.

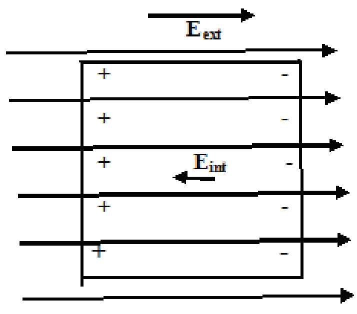
1. **Conductor:** The material which allow the electric current to pass through them, are called conductor.

   Example: Metals, human body, electrolytes etc.

2. **Insulator :** The material which do not allow electric current to pass through them, are called insulator.

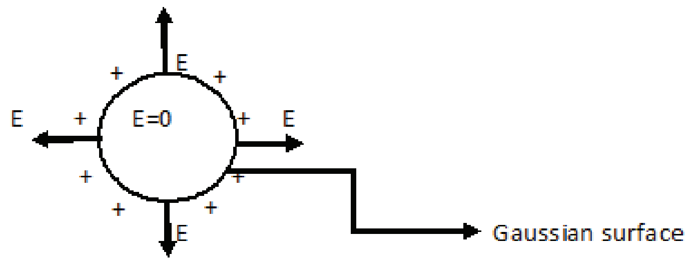
   Example: glass, wood ,mica, wax etc.

Behaviour of conductors in electrostatic fields



1. Net electrostatic field is zero in the interior of a conductor.
2. Just outside the surface of charged conductor, electric field is normal to the surface.
3. The net charge in the interior of a conductor is zero and any excess charge resides at its

surface.



4. Potential is constant within and on the surface of a conductor.
5. Electric field at the surface of a charged conductor is proportional to the surface charge density.
6. Electric field is zero in the cavity of a hollow charged conductor.

DIELECTRIC: Dielectric are insulating material which transmit electric effect without actually conducting itself. Example: Mica, ceramics etc.

There are two types of dielectric -

1. **Non polar dielectric**: These are the dielectrics in which the center of positive charge coincides with the center of negative charge is called non polar dielectric. Example: H_2, N_2, O_2 etc.
2. **Polar dielectric :** These are the dielectrics in which the center of positive charge do not coincide with the center of negative charge is called polar dielectric. Example: H_2O, HCl etc.

DIELECTRIC CONSTANT (K) : It can be regarded as the ratio of absolute permittivity of medium to that of free space is called dielectric constant ($K\ or\ \varepsilon_r$).

$$K = \varepsilon_r = \frac{\varepsilon_0}{\varepsilon}$$

DIELECTRIC POLARIZATION: Dielectric Polarization occurs when an external electric field is applied to a dielectric substance. When an electric field is applied, it causes charges (both positive and negative) to be displaced.

POLARIZATION DENSITY: The induced dipole moment developed per unit volume of a dielectric when placed in an external electric field is called polarization density.

$$P = \frac{Dipole\ moment\ of\ dielectric}{volume\ of\ dielcetric}$$

$$P = \frac{Qd}{Ad} = \frac{Q}{A} = \sigma_p$$

ELECTRIC SUSCEPTIBILITY: The ratio of the polarization to $\boldsymbol{\varepsilon_0}$ times the electric field is called the electric susceptibility of the dielectric.

The unit of electric susceptibility is $\mathbf{C^2/Nm^2}$

DIELECTRIC STRENGTH: The maximum electric field that can exist in a dielectric without causing the breakdown of its insulating property is called dielectric strength of the material.

The Unit of dielectric strength is **V/m**.

NOTE : 1. Liquid crystal Displays use dielectrics.

2. The dielectric material is used as an insulator and as a cooling agent in a transformer

***CAPACITOR AND CAPACITANCE (C):-**

Capacitor: A device to store charges & electrostatic potential energy.

Capacitance: Ratio of charge & potential difference. (It is Scalar) $C = \frac{Q}{V}$.

SI. unit : **farad (F)**

Capacitance of a parallel plate capacitor with no medium between plates : $C_0 = C = \frac{\epsilon_0 A}{d}$

Capacitance of a parallel plate capacitor with a dielectric medium of dielectric constant K in between :

Thicness = t $C_m = \frac{\epsilon_0 A}{\left(d - t + \frac{t}{K}\right)}$

Thicness = t=0 $C_0 = \frac{\varepsilon_0 A}{d}$

Thicness = t=d $C_m = K\frac{\epsilon_0 A}{d}$

$\Rightarrow \quad C_m = KC_0$

***Combination of capacitors**: (i) Capacitors in series: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

(ii) Capacitors in parallel : $C = \sum_{i=1}^{n} C_i$

***Energy stored in capacitors**: $U = \frac{1}{2} CV^2 = \frac{1}{2} QV = \frac{Q^2}{2C}$

***Energy density** : $U_d = \frac{1}{2} \epsilon_0 E^2 = \frac{\sigma^2}{2\epsilon_0}$

***Introducing dielectric slab between the plates of the charged conductor with:**

PROPERTY	BATTERY CONNECTED	BATTERY DISCONNECTED
Charge	KQ_0	Q_0
Potential difference	V_0	V_0/K
Electric Field	E_0	E_0/K
Capacitance	KC_0	KC_0
Energy	$K \frac{1}{2}\epsilon_0 E^2$ (Energy is supplied by battery)	$\frac{1}{K}\frac{1}{2}\epsilon_0 E^2$ (Energy used for polarization)

***On connecting two charged capacitors:**

(a) Common Potential : $V = \frac{C_1V_1+C_2V_2}{V_1+V_2}$

(b)Loss of energy : $\Delta U = \frac{1}{2}\frac{C_1 \times C}{(C_1+C_2)}(V_1 - V_2)^2$

SOME USEFUL LINKS:

https://www.learncbse.in/important-questions-for-class-12-physics-chapter-2/

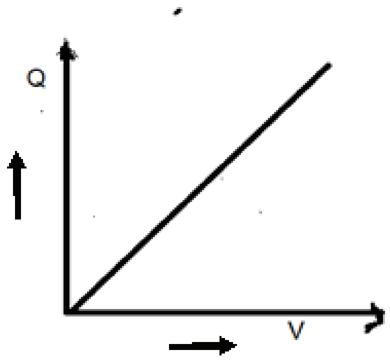
https://www.learncbse.in/ncert-exemplar-problems-class-12-physics-electrostatic-potential-capacitance/

https://phet.colorado.edu/sims/html/capacitor-lab-basics/latest/capacitor-lab-basics_en.html

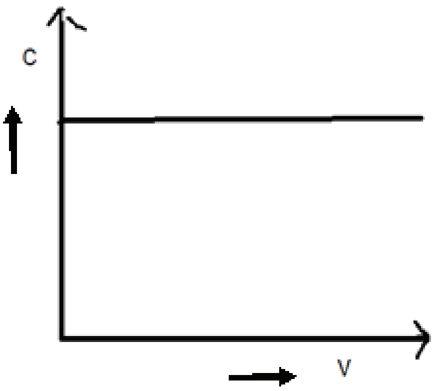
https://ophysics.com/em5.html

GRAPHS

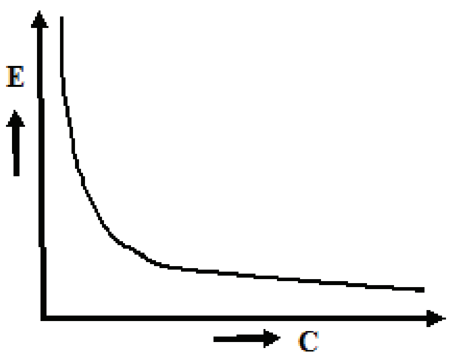
1.Graph between Q &V



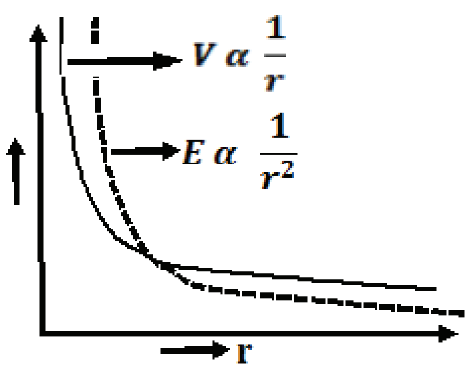
2.Graph between C& V



3.Graph between E & C



4. Graph of E & V verses distance r for a point charge



Formulae and Units:

Potential difference : work done / charge =W/q

*Electric potential due to point charge q at a distance r from it : V =Kq /r (1 /4π∈0 = K)

* Electric potential at a point due to N point charges:

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}+\frac{q_3}{r_3}+-------+\frac{q_n}{r_n}\right)$$

. Electric potential at a point due to a dipole:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r}$$

Potential Energy of a system of two-point charges: $U = \frac{q_1 q_2}{4\pi\epsilon_0 r}$

Potential Energy of a system of three-point charges: $U = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{r_{12}}+\frac{q_1 q_3}{r_{13}}+\frac{q_2 q_3}{r_{23}}\right]$

Potential energy of a single charge in an external Field: U (**r**) = q V (**r**)

Potential energy of two charges in an external Field U (**r**) = q_1V (**r₁**) +q_1V (**r₁**) + $\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}}$

Electric Potential Energy of an electric dipole

U= pE($\cos\theta_1$ - $\cos\theta_2$)

if $\theta_1 = 90^o$ *and* $\theta_2 = \theta$ then U= -pE$\cos\theta$ = -$\vec{p}.\vec{E}$

Units: Charge- coulomb, Electric dipole moment- coulomb metre (Cm)

Distance- metre ,

Energy- joule or electron volt (eV) (1eV = 1.6 × 10^{-19} J)