Ch – 1 Relations and Functions

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

Relation

A Relation R from a non-empty set A to a non-empty set B is a subset of the Cartesian product set A × B. i.e. R $\subset$ A × B. A relation in a set A is the subset of Cartesian product AXA. If a and b are elements of set A and a is related to b then we write as (a,b) ∈ R or aRb. The set of all first elements in the ordered pair (a,b) in a relation R, is called the domain of the relation R and the set of all second elements called images, is called the range of R.

Types of relations

Empty relation: A relation R in a set A is called *empty relation*, if no element of A is related to any element of A. i.e., R = $\phi \subset A \times A$.

Universal relation: A relation R in a set A is called *universal relation*, if each element of A is related to every element of A, i.e. R = A × A.

Equivalence relation. A relation R in a set A is said to be an *equivalence relation* if R is reflexive, symmetric and transitive

A relation R in a set A is called

(i) ***Reflexive***, if *(a, a)* ∈ R, for every *a* ∈ A,

(ii) ***Symmetric***, if *(a, b)* ∈ R implies that *(a, b)* ∈ R, for all *a, b* ∈ A.

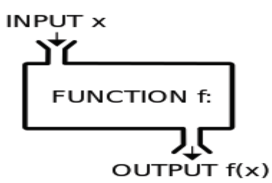
(iii) ***Transitive***, if *(a, b)*∈ R and *(b, c)* ∈ R implies that *(a, c)* ∈ R *a, b, c* ∈ A.

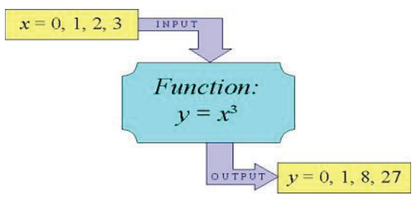
Function

A relation f from a set A to a set B is said to be function if every element of set A has one and only one image in set B. The notation f : X →Y means that f is a function from X to Y. X is called the domain of f and Y is called the co-domain of f.

Given an element x ∈ X, there is a unique element y in Y that is related to x. The unique element y to which f relates x is denoted by f (x) and is called f of x, or the value of f at x, or the image of x under f. The set of all values of f(x) taken together is called the range of f or image of X under f.

Symbolically, Range of f = {y ∈ Y | y = f (x), for some x in X}.





Types of functions:

***One-one* (or *injective*):**

A function *f*: X → Y is defined to be *one-one* (or *injective*), if the images of distinct elements of X under *f* are distinct,

i.e., for every *a, b in* X, *f* (*a*) = *f* (b) implies *a* = *b*. Otherwise, *f* is called *many one*

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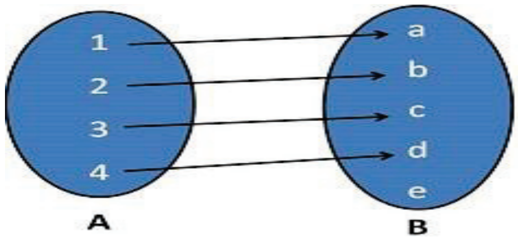
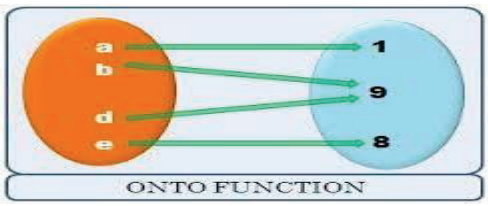
To disprove the result, we need to use its contrapositive statement:$a \neq b \Longrightarrow f(a) \neq f(b)$.



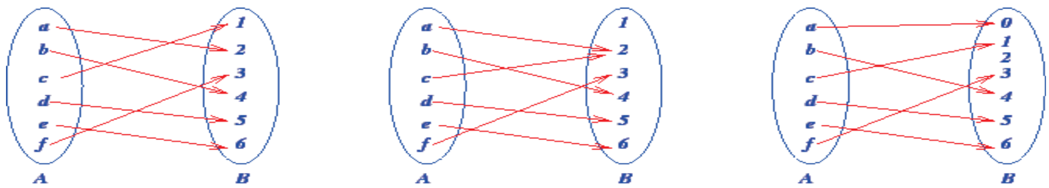
Onto **(or** ***surjective*****):**

A function f : X $\rightarrow$Y is said to be *onto* (or *surjective*), if every element of Y is the image of some element of X under f, i.e., for every y *in* Y, there exists an element x in X such that $f(x) = y$.
OR $Co - domf = Rangef$



Bijective function:

A function f : X→ Y is said to be *bijective* if f is both one-one and onto.



(p) a bijection (q) not a bijection (r) not a bijection

Important points on Relation & Functions:

1. *The total number of relations from a set A having 'm' elements to a set B having 'n' elements is* 2^{mn}.
2. *The total number of relations in a set A having 'n' elements is* 2^{n^2}.
3. *The number of reflexive relations in a set having n elements is* 2^{n^2-n}.
4. *The number of symmetric relations in a set having n elements is* $2^{\frac{n^2+n}{2}}$.
5. *The total number of equivalence relations in a set A having 'n' elements:*
   By Bell numbers

n	***1***
1	*1* ***2***
2	*2* *3* ***5***
3	*5* *7* *10* ***15***

4	*15 20 27 37* ***52***
	and so on

6. *The number of injective/one to one function(s) from a set A having 'm' to another set B having 'n' elements with* $n \geq m$ *is* $\frac{m!}{(m-n)!}$.
7. *The number of injective/one to one function(s) from a set A having 'm' to another set B having 'n' elements is* n^m.
8. *The number of onto functions from a set A having 'm' elements to a set B having 'n' elements* $m \geq n$ *is* $2^m - 2$
9. *The number of bijective functions in a set A having 'n' elements is n!*

MULTIPLE COICE QUESTIONS (1 MARK EACH)

1. If $A = \{5,6,7\}$ and let R = {(5,5), (6,6)), (7,7), (5,6), (6,5), (6,7), (7,6)}. Then R is
   A) Reflexive, symmetric but not Transitive
   B) Symmetric, transitive but not reflexive
   C) Reflexive, Transitive but not symmetric
   D) an equivalence relation
2. Let R be a relation defined on Z as follows:
   $(a, b) \in R$ If $a^2 + b^2 = 25. Then\ Domain\ of\ R$ is
   A) {3,4,5}
   B) {0,3,4,5}
   C) {0, ±3, ±4, ±5}
   D) None of these
3. The maximum number of equivalence relations on the set A= {1, 2, 3} is
   A) 1
   B) 2
   C) 3
   D)5
4. The number of elements in set A is 3. The number of possible relations that can be defined on A is
   A)8
   B)4
   C)64
   D)512
5. The number of elements in Set A is 3. The number of possible reflexive relations that can be defined in A is
   A) 64
   B) 8
   C)512
   D) 4