ALGEBRA

POLYNOMIALS

IMPORTANT CONCEPTS

- **A polynomial is an algebraic expression in which the exponent on any variable is a wholenumber. / A polynomial is an algebraic expression with variables having positive integralpowers only.**
- General Form:

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

- **Degree of a polynomial**

- The highest power of x in p(x) is called the degree of the polynomial p(x).

Name of the polynomial	Degree of the polynomial	Example
Zero polynomial	Not defined	0,5,-3…..
Linear polynomial	1	x-3
Quadratic polynomial	2	$6x^2$-3y
Cubic polynomial	3	$4x^3+5y^2-1$

❖ Value of a polynomial:

If p(x) is a polynomial in x, and if k is any real number, then the value obtained byreplacing x by k in p(x), is called the value of p(x) at x = k, and is denoted by p(k).

Q. Find the value of the polynomial $p(x) = x^2 + 4x + 4$ where x = 2.
Given polynomial: $p(x) = x^2 + 4x + 4$.
Value of given polynomial when x = 2 and we get: $p(2) = (2)^2 + 4(2) + 4$
$= 4 + 8 + 4 = 16$
Hence the value of $p(x) = x^2 + 4x + 4$, where x = 2, is 16

❖ Zero of a polynomial

A real number k is said to be a zero of a polynomial p(x), if p(k) = 0

What is the value of $p(x) = x^2 - 3x - 4$ at $x = -1$? We have : $p(-1) = (-1) 2 - \{3 \times (-1)\} - 4 = 0$ Also, note that $p(4) = 4^2 - (3 \times 4) - 4 = 0$. As $p(-1) = 0$ and $p(4) = 0$, -1 and 4 are called the zeroes of the quadratic polynomial $x^2 - 3x - 4$.

RELATIONSHIP BETWEEN ZEROES & COEFFICIENTS OF POLYNOMIALS

Type of Polynomial	General form	No. of zeroes	Relationship between zeroes and coefficients
Linear	$ax + b, a \neq 0$	1	$k = -\frac{b}{a}$, i.e. $k = -\frac{\text{Constant term}}{\text{Coefficient of x}}$
Quadratic	$ax^2 + bx + c, a \neq 0$	2	Sum of zeroes $(\alpha + \beta) = -\frac{\text{Coefficient of x}}{\text{Coefficient of x}^2} = -\frac{b}{a}$ Product of zeroes $(\alpha\beta) = \frac{\text{Constant term}}{\text{Coefficient of x}^2} = \frac{c}{a}$
Cubic	$ax^3 + bx^2 + cx + d$, $a \neq 0$	3	Sum of zeroes $(\alpha + \beta + \gamma) = -\frac{\text{Coefficient of x}^2}{\text{Coefficient of x}^3} = -\frac{b}{a}$ Product of sum of zeroes taken two at a time $(\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{\text{Coefficient of x}}{\text{Coefficient of x}^3} = \frac{c}{a}$ Product of zeroes $(\alpha\beta\gamma) = -\frac{\text{Constant term}}{\text{Coefficient of x}^3} = -\frac{d}{a}$