CHAPTERWISE QUESTION ANSWERS

Class X **MATHEMATICS**

POLYNOMIAL

SET A

SECTION - A

1. a) 2
2. a) both cannot be positive
3. c) $\sqrt{3}, \frac{-7}{\sqrt{3}}$
4. d) a = 0, b = –6
5. b) $x^2 - 8x - 9$

OR

c) –16

6. b) $\frac{c}{a}$
7. b) p = r = –2
8. a) only (ii)
9. c) Assertion (A) is true but reason (R) is false.
10. d) Assertion (A) is false but reason (R) is true.

SECTION - B

11. p = 5, q = –6
12.

Let $f(x) = 2x^2 + \frac{7}{2}x + \frac{3}{4} = 8x^2 + 14x + 3$

$= 8x^2 + 12x + 2x + 3$ [by splitting the middle term]

$= 4x(2x + 3) + 1(2x + 3)$

$= (2x + 3)(4x + 1)$

So, the value of $8x^2 + 14x + 3$ is zero when $2x + 3 = 0$ or $4x + 1 = 0$,

i.e., when $x = -\frac{3}{2}$ or $x = -\frac{1}{4}$.

So, the zeroes of $8x^2 + 14x + 3$ are $-\frac{3}{2}$ and $-\frac{1}{4}$.

$\therefore$ Sum of zeroes $= -\frac{3}{2} - \frac{1}{4} = -\frac{7}{4} = \frac{-7}{2 \times 2}$

$= -\frac{(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$

And roduct of zeroes $= \left(-\frac{3}{2}\right)\left(-\frac{1}{4}\right) = \frac{3}{8} = \frac{3}{2 \times 4}$

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

OR

Let the roots be α and $\frac{1}{\alpha}$. Then $\alpha\left(\frac{1}{\alpha}\right) = \frac{m}{7}$ or $1 = \frac{m}{7}$ or m = 7

SECTION - C

13. Let one zero of the given polynomial be α

Then, the other zero is $\frac{1}{\alpha}$

$\therefore$ Product of zeroes = $\alpha \times \frac{1}{\alpha} = 1$

But, as per the given polynomial product of zeroes = $\frac{6a}{a^2 + 9}$

$\therefore \quad \frac{6a}{a^2 + 9} = 1$

$\Rightarrow a^2 + 9 = 6a$

$\Rightarrow \quad a^2 - 6a + 9 = 0 \qquad \Rightarrow (a - 3)^2 = 0$

$\Rightarrow \quad a - 3 = 0 \qquad \Rightarrow a = 3$

Hence, a = 3.

14.

Given quadratic polynomial is

$f(x) = x^2 - 4x + 3$

$\therefore$ Sum of zeros $= \alpha + \beta = \frac{-(-4)}{1} = 4$

$\Rightarrow \quad \alpha + \beta = 4$

and product of zeros, $\alpha\beta = \frac{3}{1} = 3$

$\because \quad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow \quad \alpha^2 + \beta^2 = (4)^2 - 2 \times 3 = 16 - 6 = 10$

Now, $\alpha^4\beta^2 + \alpha^2\beta^4 = \alpha^2\beta^2(\alpha^2 + \beta^2) = (3)^2 \times 10 = 9 \times 10 = 90$

$\therefore \quad \alpha^4\beta^2 + \alpha^2\beta^4 = 90$

OR

Let α and β be the zeros of the polynomial. Then as per question $\beta = 7\alpha$

Now sum of zeros $= \alpha + \beta = \alpha + 7\alpha = -\left(\frac{-8}{3}\right)$

$\Rightarrow \quad 8\alpha = \frac{8}{3} \quad$ or $\quad \alpha = \frac{1}{3}$

and $\quad \alpha \times \beta = \alpha \times 7\alpha = \frac{2k+1}{3}$

$\Rightarrow \quad 7\alpha^2 = \frac{2k+1}{3} \quad \Rightarrow \quad 7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3}$

$\Rightarrow \quad \frac{7}{9} = \frac{2k+1}{3} \quad \Rightarrow \quad \frac{7}{3} = 2k+1$

$\Rightarrow \quad \frac{7}{3} - 1 = 2k \quad \Rightarrow \quad k = \frac{2}{3}$

15. k = 0

16. Writes the given polynomial as :

$p(x) = x^2 + 9 + 4x + 2c$

Writes that, if p(x) is divisible by x, p(0) = 0

OR

Writes that the remainder of $\frac{p(x)}{x}$, which is 9 + 2c, should be 0.

Finds the value of c as $\frac{-9}{2}$

SECTION - D

17. Let α, β, γ are the zeroes of f(x). If α, β, γ are in AP, then,

$\beta - \alpha = \gamma - \beta \quad \Rightarrow \quad 2\beta = \alpha + \gamma$

$\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(-12)}{1} = 12 \quad \Rightarrow \quad \alpha + \gamma = 12 - \beta$

From (*i*) and (*ii*)

$2\beta = 12 - \beta \quad$ or $\quad 3\beta = 12 \quad$ or $\quad \beta = 4$

Putting the value of β in (*i*), we have

$8 = \alpha + \gamma$

$\alpha\beta\gamma = -\frac{d}{a} = \frac{-(-28)}{1} = 28$

$(\alpha\gamma)\,4 = 28 \quad$ or $\quad \alpha\gamma = 7 \quad$ or $\quad \gamma = \frac{7}{\alpha}$

Putting the value of $\gamma = \frac{7}{\alpha}$ in (*iii*), we get

$\Rightarrow \quad 8 = \alpha + \frac{7}{\alpha} \quad \Rightarrow \quad 8\alpha = \alpha^2 + 7$

$\Rightarrow \quad \alpha^2 - 8\alpha + 7 = 0 \quad \Rightarrow \quad \alpha^2 - 7\alpha - 1\alpha + 7 = 0$

$\Rightarrow \quad \alpha(\alpha - 7) - 1\,(\alpha - 7) = 0 \quad \Rightarrow \quad (\alpha - 1)(\alpha - 7) = 0$

$\Rightarrow \quad \alpha = 1 \quad$ or $\quad \alpha = 7$

Putting $\alpha = 1$ in (*iv*), we get

$$\gamma = \frac{7}{1}$$

or $\gamma = 7$

and $\beta = 4$

$\therefore$ zeros are 1, 7, 4.

Putting $\alpha = 7$ in (*iv*), we get

$$\gamma = \frac{7}{7}$$

or $\gamma = 1$

and $\beta = 4$

$\therefore$ zeros are 7, 4, 1.

Hence zeros are 1, 4, 7 or 7, 4, 1

18.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we get $a = 3, b = -5, c = -11, d = -3$. Further

$p(3) = 3 \times 3^3 - (5 \times 3^2) - (11 \times 3) - 3 = 81 - 45 - 33 - 3 = 0,$

$p(-1) = 3 \times (-1)^3 - 5 \times (-1)^2 - 11 \times (-1) - 3 = -3 - 5 + 11 - 3 = 0,$

$$p\left(-\frac{1}{3}\right) = 3 \times \left(-\frac{1}{3}\right)^3 - 5 \times \left(-\frac{1}{3}\right)^2 - 11 \times \left(-\frac{1}{3}\right) - 3,$$

$$= -\frac{1}{9} - \frac{5}{9} + \frac{11}{3} - 3 = -\frac{2}{3} + \frac{2}{3} = 0$$

Therefore, 3, -1 and $-\frac{1}{3}$ are the zeroes of $3x^3 - 5x^2 - 11x - 3$.

So, we take $\alpha = 3$, $\beta = -1$ and $\gamma = -\frac{1}{3}$.

Now,

$$\alpha + \beta + \gamma = 3 + (-1) + \left(-\frac{1}{3}\right) = 2 - \frac{1}{3} = \frac{5}{3} = \frac{-(-5)}{3} = \frac{-b}{a},$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 3 \times (-1) + (-1) \times \left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right) \times 3 = -3 + \frac{1}{3} - 1 = \frac{-11}{3} = \frac{c}{a},$$

$$\alpha\beta\gamma = 3 \times (-1) \times \left(-\frac{1}{3}\right) = 1 = \frac{-(-3)}{3} = \frac{-d}{a}.$$

OR

Let $p(x) = x^3 - 4x^2 + 5x - 2$

On comparing with general polynomial $p(x) = ax^3 + bx^2 + cx + d$, we get $a = 1, b = -4, c = 5$ and $d = -2$.

Given zeros 2, 1, 1.

$\therefore \quad p(2) = (2)^3 - 4(2)^2 + 5(2) - 2 = 8 - 16 + 10 - 2 = 0$

and $\quad p(1) = (1)^3 - 4(1)^2 + 5(1) - 2 = 1 - 4 + 5 - 2 = 0.$

Hence, 2, 1 and 1 are the zeros of the given cubic polynomial.

Again, consider $\alpha = 2, \beta = 1, \gamma = 1$

$\therefore \quad \alpha + \beta + \gamma = 2 + 1 + 1 = 4$

and $\quad \alpha + \beta + \gamma = \dfrac{-(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3} = \dfrac{-b}{a} = \dfrac{-(-4)}{1} = 4$

$\alpha\beta + \beta\gamma + \gamma\alpha = (2)(1) + (1)(1) + (1)(2) = 2 + 1 + 2 = 5$

and $\quad \alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \dfrac{c}{a} = \dfrac{5}{1} = 5$

$\alpha\beta\gamma = (2)(1)(1) = 2$

and $\quad \alpha\beta\gamma = \dfrac{-(\text{Constant term})}{\text{Coefficient of } x^3} = \dfrac{-d}{a} = \dfrac{-(-2)}{1} = 2$

SECTION - E

CASE STUDY

19. i) $-4, 7$ ii) $-x^2 + 3x + 28$ **OR** -28 iii) -35

SET B

1. b) -1
2. d) 3
3. a) $a = \dfrac{1}{2}$, $c = 6$

OR

b) 3 and -2

4. c) $k(x^2 - 6x + 4)$
5. a) $\dfrac{4}{3}$
6. d)
7. b) $9x^2 + 82x + 9$
8. a) 3/2

9. b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

10. a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

SECTION - B

11. Since – 4 is a zero of the given polynomial

$\therefore$ $(k + 2)(-4)^2 + k(-4) + 4 = 0$

$\Rightarrow$ $16k + 32 - 4k + 4 = 0$

$12k = -36$

$k = -3$

12. Let α and β be the zeroes of the quadratic polynomial.

$\therefore$ Sum of zeroes, $\alpha + \beta = -1$ and product of zeroes, $\alpha . \beta = -20$

Now, quadratic polynomial be $x^2 - (\alpha + \beta). x + \alpha\beta$

$= x^2 - (-1)x - 20 = x^2 + x - 20$

Now, for zeroes of this polynomial

$x^2 + x - 20 = 0$

$\Rightarrow x(x + 5) - 4(x + 5) = 0$ $\qquad$ $\Rightarrow x^2 + 5x - 4x - 20 = 0$

$\Rightarrow x = -5, 4$ $\qquad$ $\Rightarrow (x + 5)(x - 4) = 0$

$\therefore$ zeroes are –5 and 4.

OR

Let $f(t) = t^3 - 2t^2 - 15t$

$= t(t^2 - 2t - 15)$

$= t(t^2 - 5t + 3t - 15)$ [by splitting the middle term]

$= t[t(t - 5) + 3(t - 5)]$

$= t(t - 5)(t + 3)$

So, the value of $t^3 - 2t^2 - 15t$ is zero when $t = 0$ or $t - 5 = 0$ or $t + 3 = 0$

i.e., when $t = 0$ or $t = 5$ or $t = -3$.

So, the zeroes of $t^3 - 2t^2 - 15t$ are –3, 0 and 5.

$\therefore$ Sum of zeroes $= -3 + 0 + 5 = 2 = \frac{-(-2)}{1}$

$= (-1) . \left(\frac{\text{Coefficient of } t^2}{\text{Coefficient of } t^3}\right)$

Sum of product of two zeroes at a time

$= (-3)(0) + (0)(5) + (5)(-3)$

$= 0 + 0 - 15 = -15$

$= (-1)^2 . \left(\frac{\text{Coefficient of } t}{\text{Coefficient of } t^3}\right)$

and product of zeroes = (–3) (0) (5) = 0

$$= (-1)^3 \left(\frac{\text{Constant term}}{\text{Coefficient of } t^3}\right)$$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

SECTION - C

13. k = –1, $\frac{2}{3}$

14. a = $\frac{1}{2}$ and c = 5

OR

Since α and β are the zeros of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$

$$\therefore \quad \alpha + \beta = \frac{-(-5)}{2} = \frac{5}{2} \quad \text{and} \quad \alpha\beta = \frac{7}{2}$$

Let S and P denote respectively the sum and product of the zeros of the required polynomial.

Then, $S = (2\alpha + 3\beta) + (3\alpha + 2\beta) = 5(\alpha + \beta) = 5 \times \frac{5}{2} = \frac{25}{2}$

and $P = (2\alpha + 3\beta)(3\alpha + 2\beta)$

$$\Rightarrow \quad P = 6\alpha^2 + 6\beta^2 + 13\alpha\beta = 6\alpha^2 + 6\beta^2 + 12\alpha\beta + \alpha\beta$$

$$= 6(\alpha^2 + \beta^2 + 2\alpha\beta) + \alpha\beta = 6(\alpha + \beta)^2 + \alpha\beta$$

$$\Rightarrow \quad P = 6 \times \left(\frac{5}{2}\right)^2 + \frac{7}{2} = \frac{75}{2} + \frac{7}{2} = 41$$

Hence, the required polynomial $g(x)$ is given by

$$g(x) = k(x^2 - Sx + P)$$

or $g(x) = k\left(x^2 - \frac{25}{2}x + 41\right)$, where k is any non-zero real number.

15. k= 12

16. Writes the equation for the product of zeroes as :

$$\left(\frac{3+\sqrt{k}}{2}\right)\left(\frac{3-\sqrt{k}}{2}\right) = \frac{-3}{2}$$

Simplifies the above equation and writes : $\frac{9-k}{4} = \frac{-3}{2}$

Solves the above equation and finds the value of k as 15.

SECTION - D

17.

Let the cubic polynomial be $p(x) = ax^3 + bx^2 + cx + d$. Then

Sum of zeros $= \frac{-b}{a} = 2$

Sum of the products of zeros taken two at a time $= \frac{c}{a} = -7$

and product of the zeros $= \frac{-d}{a} = -14$

$\Rightarrow \quad \frac{b}{a} = -2, \quad \frac{c}{a} = -7, \quad -\frac{d}{a} = -14 \quad$ or $\quad \frac{d}{a} = 14$

$\therefore \quad p(x) = ax^3 + bx^2 + cx + d \quad \Rightarrow \quad p(x) = a\left[x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a}\right]$

$p(x) = a\{x^3 + (-2)x^2 + (-7)x + 14] \quad \Rightarrow p(x) = a[x^3 - 2x^2 - 7x + 14]$

For real value of $a = 1$, $p(x) = x^3 - 2x^2 - 7x + 14$

OR

Let α, β and γ be the zeros of polynomial $f(x)$ such that $\alpha\beta = 12$.

We have, $\quad \alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(-5)}{1} = 5$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{-2}{1} = -2 \quad$ and $\quad \alpha\beta\gamma = \frac{-d}{a} = \frac{-24}{1} = -24$

Putting $\alpha\beta = 12$ in $\alpha\beta\gamma = -24$, we get

$12\gamma = -24 \quad \Rightarrow \quad \gamma = -\frac{24}{12} = -2$

Now, $\alpha + \beta + \gamma = 5 \quad \Rightarrow \quad \alpha + \beta - 2 = 5$

$\Rightarrow \quad \alpha + \beta = 7 \quad \Rightarrow \quad \alpha = 7 - \beta$

$\Rightarrow \quad (7 - \beta)\beta = 12 \quad [\because \alpha\beta = 12] \quad \Rightarrow \quad 7\beta - \beta^2 = 12$

$\Rightarrow \quad \beta^2 - 7\beta + 12 = 0 \quad \Rightarrow \quad \beta^2 - 3\beta - 4\beta + 12 = 0$

$\Rightarrow \quad \beta(\beta - 3) - 4(\beta - 3) = 0 \quad \Rightarrow \quad (\beta - 4)(\beta - 3) = 0 \quad \Rightarrow \quad \beta = 4 \quad$ or $\quad \beta = 3$

$\therefore \quad \alpha = 3 \quad$ or $\quad \alpha = 4$

So, zeroes of $f(x)$ are 3, 4, – 2

18. Sum of zeroes = a + b = p

Product of zeroes = ab = q

$$\frac{a^2}{b^2} + \frac{b^2}{a^2} = \frac{a^4 + b^4}{a^2b^2} = \frac{(a^2 + b^2)^2 - 2a^2b^2}{a^2b^2}$$

$$= \frac{[(a+b)^2 - 2ab]^2 - 2a^2b^2}{a^2b^2} = \frac{[p^2 - 2q]^2 - 2q^2}{q^2}$$

$$= \frac{p^4 - 4p^2q + 4q^2 - 2q^2}{q^2} = \frac{p^4 - 4p^2q + 2q^2}{q^2}$$

$$= \frac{p^4}{q^2} - \frac{-4p^2q}{q^2} + \frac{2q^2}{q^2}$$

$$= \frac{p^4}{q^2} - \frac{-4p^2q}{q^2} + 2$$

SECTION - E

CASE STUDY

19. i) No, coordinates are (0, –2)

    ii) $P(y) = 0.25\ y^3 + 0.1\ y^2 - 0.3\ y + 1$

    iii) The curves of the flower pot can be different.

OR

Change X axis interceptions

Change coinstant term

Change the points where polynomials cuts the X axis

Decrease 1 and increase –1.