RELATIONS AND FUNCTIONS

MULTIPLE CHOICE QUESTIONS

Q.No	QUESTIONS AND SOLUTIONS
1	If $A = \{5,6,7\}$ and let $R = \{5,5), (6,6)), (7,7), (5,6), (6,5), (6,7), (7,6)\}$. Then R is a) Reflexive, symmetric but not Transitive b) Symmetric, transitive but not reflexive c) Reflexive, Transitive but not symmetric d) an equivalence relation **Answer: A** (5,6) ∈ R and (6,7) ∈ R but (5,7) does not belong to R
2	Let R be a relation defined on Z as follows: $(a, b) \in R \Leftrightarrow a^2 + b^2 = 25. Then\ Domain\ of\ R\ is$ a) {3,4,5} b) {0,3,4,5} c) {0,±3,±4,±5} d) None of these **Answer: C** R={(0, ,±5), (,±5,0),(±3, ,±4),(,±4, ±3)} Domain of R is the set of all first elements of R.
3	The maximum number of equivalence relations on the set $A = \{1 , 2 , 3\}$ is a) 1 b) 2 c) 3 d)5 **Answer: D** Possible equivalence relations are R_1 = { (1,1) , (2,2) , (3,3) } R_2 = { (1,1) , (2,2) , (3,3), (1,2) ,(2,1)} R_3 = { (1,1) , (2,2) , (3,3), (1,3), (3,1)} R_4 = { (1,1) , (2,2) , (3,3), (2,3),(3,2) } R_5 = A×A
4	Consider the set $A = \{1, 2\}$. The relation on A which is symmetric but neither transitive nor reflexive is a) {(1,1) (2,2)} b) { } c) {(1,2)} d) { (1,2) (2,1) } **Answer: D**

	R is not reflexive since (1,1) and (2,2) are not there in R R is not transitive since (1,2) and (2,1) belong to R but (1,1) does not belong to R.
5	If $A = \{d, \mathrm{e}, \mathrm{f}\}$ and let $\mathrm{R} = \{(\mathrm{d}, \mathrm{d}), (\mathrm{d}, \mathrm{e}), (\mathrm{e}, \mathrm{d}), (e, e)\}$. Then R is a) Reflexive, symmetric but not Transitive b) Symmetric, transitive but not reflexive c) Reflexive, Transitive but not symmetric d) an equivalence relation **Answer: B** R is not reflexive because (f, f) is not present in R
6	Let R be a reflexive relation on a finite set A having n elements and let there be m,minimum number of ordered pairs in R, then a) m < n b) m > n c) m = n d)none of these **Answer: C** A relation on a set A is reflexive if every element of A is related to itself i.e. $(a, a) \in R, for\ all\ a \in R$
7	The number of elements in set A is 3.The number of possible relations that can be defined on A is a) 8 b) 4 c) 64 d) 512 **Answer: D** The number of possible relations on a set having n elements is 2^{n^2} as every relation is a subset of A× A.
8	The number of elements in Set A is 3.The number of possible reflexive relations that can be defined in A is a) 64 b) 8 c) 512 d) 4 **Answer: A** If a set has A has n elements then the number of possible reflexive relations on A is $2^{n(n-1)}$.
9	The number of elements in set P is 4.The number of possible symmetric relations that can be defined on P is a) 16 b) 32 c) 512 d) 1024 **Answer: D** If a set has A has n elements then the number of possible symmetric relations on A is $2^{\frac{n(n+1)}{2}}$
10	Let R be a relation on the set N of natural numbers defined by aRb if and only if $a\ divides\ b$.Then R is a) Reflexive, symmetric but not Transitive b) Symmetric, transitive but not reflexive c) Reflexive, Transitive but not symmetric d) an equivalence relation **Answer C** R is reflexive, since every natural number divides itself. If a divides b and b divides c then a divides c So R is transitive $a\ divides\ b$ need not imply that $b\ divides\ a$. So R is not symmetric.

EXERCISE

1.	Let $f: R \rightarrow R\ be\ defined\ by\ f(x) = \frac{1}{x}\ \forall\ x \in R. Then\ f\ is$ a) One-one b) Onto c) Bijective d) f is not defined **Answer: d**
2.	Set A has 4 elements and set B has 5 elements. Then the number of bijective mappings from A to B is a) 120 b) 20 c) 0 d) 625 **Answer: c**
3.	Set A has 3 elements and set B has 4 elements. Then the number of injective mappings from A to B is a) 144 b) 12 c) 24 d) 64 **Answer: c**
4.	The function $f: [\pi, 2\pi] \rightarrow R$ defined by $f(x) = cosx$ is a) one – one but not onto b) onto but not one – one c) many – one function d) bijective function **Answer: d**
5.	Let $f: R \rightarrow R\ be\ defined\ by\ f(x) = x^3 + 4$,then f is a) Injective b) Surjective c) Bijective d) None of these **Answer: c**

6.	Let $A = \{1,2,3\}, B = \{4,5,6,7\}$ and let $f = \{(1,4),(2,5),(3,6)\}$ be a function from A to B. Based on the given information f is best defined as a) Surjective function b) Injective function c) Bijective function d) Function **Answer: b**
7.	Let $A = \{1,2,3,\ldots,n\} and\ B = \{p.q\}$.Then the number of onto functions from A to B is a) 2^n b) $2^n - 2$ c) $2^n - 1$ d) None of these **Answer: b**

ASSERTION AND REASONING QUESTIONS

	In the following question a statement of Assertion (A) is followed by a statement of Reason(R).Pick the correct option: A) Both A and R are true and R is the correct explanation of A. B) Both A and R are true but R is NOT the correct explanation of A. C) A is true but R is false. D) A is false but R is true.
1.	**Assertion (A):** *If n (A) =p and n (B) = q then the number of relations from A to B is 2^{pq}.* **Reason(R):** *A relation from A to B is a subset of $A \times B$.* **Answer. A** **Solution:** Every relation from set A to set B is a subset of $A \times B$. So R is true The number of elements in $A \times B$ is $p \times q$.So number of subsets of $A \times B$.i.e no of relations from A to B is 2^{pq} . So A is true.
2.	**Assertion (A):** *If n (A) =m, then the number of reflexive relations on A is m* **Reason(R) :** *A relation R on the set A is reflexive if (a, a) $\in R, \forall a \in A$.* **Answer: D** **Solution:** A relation R is reflexive on the set A iff (a,a) ∈ R ∀ a ∈ A. So R is true. $n(A) = m$ then the number of reflexive relations on A is 2^{m^2-m}. So A is false.
3.	**Assertion (A):** Domain and Range of a relation $R = \{(x,y): x - 2y = 0\}$defined on the set A = $\{1,2,3,4\}$ are respectively $\{1,2,3,4\}$and $\{2,4,6,8\}$ **Reason(R):** Domain and Range of a relation R are respectively the sets $\{a: a \in A\ and\ (a,b) \in R.\}\ and\ \{b: b \in A\ and\ (a,b) \in R\}$ **Answer: D** **Solution:** Domain of a relation R is $\{x: x \in A\ and\ (x,y) \in R.\}$. Range of a relation R is $\{y: y \in A\ and\ (x,y) \in R.\}$. So R is true.

	$R = \{(2,1),(4,2)\}$ So A is false
4.	**Assertion (A):** A relation $R = \{(1,1),(1,2),(2,2),(2,3)(3,3)\}$ defined on the set $A = \{1,2,3\}$ is reflexive. **Reason(R):** A relation R on the set A is reflexive if $(a,a) \in R, \forall\, a \in A$ **Answer: A** **Solution:**A relation R on the set A is reflexive if $(x,x) \in R, \forall\, x \in A$ So R is true For , $\forall\, x \in A\ (x,x) \in R$ so R is reflexive and thus A is true. Therefore answer is A.
5.	**Assertion (A):** A relation $R = \{(1,1),(1,2),(2,2),(2,3)(3,3)\}$ defined on the set $A = \{1,2,3\}$ is symmetric **Reason(R):** A relation R on the set A is symmetric if $(a,b) \in R \Longrightarrow (b,a) \in R$ **Answer:D** **Solution:**A relation R on the set A is symmetric if $(x,y) \in R \Longrightarrow (y,x) \in R$ So,R is true $(1,2) \in R\ but\ (2,1)$ does not belong to R so R is not symmetric . So A is false
6.	**Assertion (A):** A relation $R = \{(1,1),(1,3),(1,5),(3,1)(3,3),(3,5)\}$ defined on the set $A = \{1,3,5\}$ is transitive. **Reason(R):** A relation R on the set A symmetric if $(\mathrm{a,b}) \in \mathrm{R}$ and $(\mathrm{a,c}) \in \mathrm{R} \Longrightarrow (\mathrm{a,c}) \in \mathrm{R}$ **Answer:C** **Solution:** A relation R on the set A transitive iff $(\mathrm{a,b}) \in \mathrm{R}$ and $(\mathrm{a,c}) \in \mathrm{R} \Longrightarrow (\mathrm{a,c}) \in \mathrm{R}$. So R is false As the condition of transitivity is satisfied, so A is true.
7.	**Assertion (A):** $A = \{1,2,3\}, B = \{4,5,6,7\}, f = \{(1,4),(2,5),(3,6)\}$ is a function from A to B.Then $f\ is\ one - one$ **Reason(R):** A function $f\ is\ one - one$ if distinct elements of A have distinct images in B. **Answer:A** **Solution:** A function f is one –one if distinct elements of A have distinct images in B. So R is true As distinct elements of Set A have distinct images in the set B. So, A is true Thus A and R are true and R is the correct explanation of A.
8.	**Assertion (A):** Consider the function $f: R \to R$ defined by $f(x) = x^3$. Then f is one-one **Reason(R):** Every polynomial function is one-one **Answer: C** **Solution:** Every polynomial function is not one-one as $f(x) = x^2$ is not one one. So R is false. A function f is one-one if distinct elements have distinct images So A is true Thus A is true but R is false.

9.	**Assertion (A):** If $X = \{0,1,2\}$ and the function $f: X \to Y$ defined by $f(x) = x^2 - 2$ is surjection then $Y = \{-2,-1,0,2\}$ **Reason(R):** If $f: X \to Y$ is surjective if for all $y \in Y$ there exists $x \in X$ such thtat $y = f(x)$ **Answer: D** **Solution:** A function is surjective or onto if $range = co - domain$,i.e $f: X \to Y$ is surjective if for all $y \in Y$ there exists $x \in X$ such thtat $y = f(x)$ So R is true. There is no $x\ in\ X$ such that $f(x) = 0$, so range of f is not equal to the codomain, i.e f is not surjective So, A is false. Thus A is false but R is true.
10.	**Assertion (A):** A, B are two sets such that $n(A) = m\ and\ n(B) = n$. The number of one-one functions from A to B is n_{p_m} ,if $n \geq m$ **Reason(R):** A function f is one –one if distinct elements of A have distinct images in B **Answer: A** **Solution:**A function f is one –one if distinct elements of A have distinct images in B. So R is true. For a function from set A to B is one-one iff $n(A) \leq n(B)$ So A is true.

EXERCISE

1	Assertion (A): A function $f: A \to B$, cannot be an onto function if $n(A) < n(B)$. Reason(R): A function f is onto if every element of co-domain has at least one pre-image in the domain **Answer:A**
2	Assertion (A): Consider the function $f: R \to R$ defined by $f(x) = \frac{x}{x^2+1}$ .Then f is one – one Reason(R): f(4)=4/17 and f(1/4)=4/17 **Answer: D**
3	Assertion (A): $n(A) = 5, n(B) = 5$ and $f : A \to B$ is one-one then f is bijection Reason(R): If $n(A) = n(B)$ then every one-one function from A to B is onto **Answer: A**
4	Assertion (A): Set A has 4 elements and set B has 5 elements. Then the number of bijective mappings from A to B is 5^4 Reason(R): A mapping from A to B cannot be bijective if n(A) is not equal to n(B). Answer: D
5	Assertion (A):The identity relation on a set A is an equivalence relation. Reason (R): The Universal relation on a set A is an equivalence relation. **Answer: B**

2 MARKS QUESTIONS

1.	The relation R defined by $(a,b)R(c,d) \Rightarrow\ a+d=b+c$ on the A×A. Where $A=\{1,2,3,\ldots\ldots,10\}$ is an equivalence relation. Find the equivalence class of the element (3,4). **Solution:** Let (3,4) R (a,b) on $A\times A$ where $A=\{1,2,3,\ldots\ldots.10\}$ $\Rightarrow 3+b=4+a\ \rightarrow\ b-a=1$ $[(3,4)]_R=\{(1,2).(2,3),(3,4),(4,5),(6,7),(7,8),(8,9),(9,10)\}$
2.	Let $f: N\rightarrow N$ be defined by $f(n)=\begin{cases}\frac{n+1}{2}, \text{if n is odd}\\ \frac{n}{2}, \text{if n is even}\end{cases}$ for all n ∈ N. Is the function f one-one or not ? Justify your answer. **Solution:** Given function is not one-one, because 1 and 2 have the same image. $f(1)=\frac{1+1}{2}=1, f(2)=\frac{2}{2}=1$
3.	Consider f : $R_+\rightarrow[-9,\infty)$ given by f(x) = 5x^2 + 6x – 9. Show that f is one-one. **Solution:** One-one: Let $f(x)=f(y)\ \Rightarrow 5x^2+6x-9=5y^2+6y-9$ $\Rightarrow 5(x-y)(x+y)+6(x-y)=0$ $\Rightarrow (x-y)(5x+5y+6)=0$ $\Rightarrow x=y. Since\ 5x+5y+6\neq 0\ for\ all\ x,y\in\ R_+$ $Given\ function\ is\ one-one.$
4.	Let R be the relation defined in the set $A\ =\ \{1,2,3,4,5\}$ by $R\ =\ \{(x,y): x,y\in A, x\ and\ y\ are\ either\ both\ odd\ or\ both\ even\}$. Find R in Roster form. **Solution:** $R=\{(1,1),(1,3),(1,5),(3,1),(3,3),(3,5),(5,1),(5,3),(5,5),(2,2),(2,4),(4,2),(4,4)\}$
5.	Check whether the following relation $R\ =\ \{(a,b): a\ \leq b\}$ defined on set of real numbers are reflexive and symmetric or not. **Solution:** for each $a\in R, a\leq a$ is true. Given relation is reflexive. (2,3)∈ $R\ but$ (3,2) $\notin R$ thus,for each (a,b)∈ $R\ \ \not\Rightarrow (b,a)\in R$ Hence given relation is not symmetric.
6.	Prove that the greatest integer function $f: R\rightarrow R$, given by $f(x)=[x]$, is neither one-one nor onto. **Solution:** $f(1.1)=1, f(1.3)=1, but\ 1.1\neq 1.3, \therefore\ f\ is\ not\ one-one.$ Range is set of integers only whereas codomain is set of real numbers. Range ≠ codomain , ∴ f is not onto
7.	Let A = R − {1}. If $f: A\rightarrow A$ is a mapping defined by f(x) $=\frac{x-2}{x-1}$, show that f is one-one. **Solution:** One-One: Let $f(x)=f(y)\Rightarrow\frac{x-2}{x-1}=\frac{y-2}{y-1}$ $(x-2)(y-1)=(y-2)(x-1)$ $\Rightarrow xy-x-2y+2=xy-2x-y+2$ $\Longrightarrow x-y=0$ $\Longrightarrow x=y$ $Given\ function\ is\ one-one.$
8.	Let A = R − {1}. If $f: A\rightarrow A$ is a mapping defined by f(x) $=\frac{x-2}{x-1}$, show that f is onto. **Solution:** $Onto$: Let $\frac{x-2}{x-1}=y\ \ \Longrightarrow x-2=y(x-1)$

	$\Longrightarrow x - 2 = xy - y$ $x(1-y) = 2 - y$ $x = \frac{2-y}{1-y}$ $for\ each\ y \in A, there\ exist\ x = \dfrac{2-y}{1-y} \in A\ such\ that\ f(x) = y.$ $Given\ function\ is\ onto.$
9.	A function $f: A \to B$ defined as $f(x) = 2x$, is both one-one and onto.If $A = \{1,2,3,4\}$,then find the set B. **Solution:** $f = \{(1,2),(2,4),(3,6),(4,8)\}$. Range of $f = \{2,4,6,8\}$. As function is onto $range\ = codomain$. So $B = \{2,4,6,8\}$.
10.	Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1, L_2): L_1$ is perpendicular to $L_2\}$. Is the relation R transitive? Justify your answer. **Solution:** R is not transitive Let $(L_1, L_2) \in R\ and\ (L_2, L_3) \in R$ $\Rightarrow L_1\ is\ perpendicular\ to\ L_2\ and\ L_2\ is\ perpendicular\ to\ L_3$ $\Rightarrow L_1 is\ parallel\ to\ L_3 \Rightarrow L_1\ is\ not\ perpendicular\ to\ L_3$ $\Rightarrow (L_1, L_3) \notin R$ Hence, R is not transitive.

EXERCISE

1.	Is the relation R = {(a, b): a$\leq b^2$} defined on set of real numbers transitive? Justify your answer. **Answer:** No, it is not transitive (5,3) ∈ R and (3,2) ∈ R but (5,2) ∉ R.
2.	Determine whether the relation R defined on the set of all real numbers as R = {(a, b): a, b and a – b + $\sqrt{3}$ ∈ S, where S is the set of all irrational numbers}, is symmetric. **Answer:** $(4\sqrt{3}, 3\sqrt{3})$ ∈ R but $(3\sqrt{3}, 4\sqrt{3})$ ∉ R. So, R is not symmetric.
3.	Is the relation R defined on the set of all real numbers as $R = \{(a,b): a > b\}$, reflexive,symmetric and trasitive? Justify your answer. **Answer:** $(a,a) \notin R$ .So, R is not reflexive. Let $(a,b) \in R \Longrightarrow a > b, but\ b\ is\ not\ greater\ than\ a \Rightarrow (b,a) \notin R$ Therefore R is not symmetric. $a > b\ and\ b > c \Longrightarrow a > c$ Therefore R is transitive.
4.	Show that the relation R defined on the set A of all triangles in a plane as $R = \{(T_1, T_2): T_1\ is\ similar\ to\ T_2\}$ is an equivalence relation.
5.	Show that the relation R in the set of ℝ real numbers, defined as $R = \{(a,b): a \leq b^3\}$ is not transitive. **Answer:** (9,4) ∈ R and (4,2) ∈ R but (9,2) ∉ R .Hence R is not transitive

3 MARKS QUESTIONS

1.	Show that the relation R in the set $\{1, 2, 3\}$ given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive. **Solution:** Let A = {1, 2, 3} The relation R is defined on A is given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} Now, we have to show that R is reflexive but neither symmetric nor transitive. <u>Reflexive</u>**:** Clearly, $(a, a) \in R\ for\ every\ a \in A$ Hence, R is reflexive <u>Symmetric:</u> Clearly, $(1, 2) \in R,\ \ but\ (2, 1) \notin R$ Thus, for every $(a, b) \in R,\ \ (b, a) \notin R$ Hence, R is **not** symmetric <u>Transitive:</u> For (1, 2) $\in R$ and (2, 3) $\in R$ $\Rightarrow$ (1, 3)$\notin R$ Thus, $(a, b) \in R\ and\ (b, c) \in R\ then\ (a, c)\ ) \notin R$ Hence, R is **not** transitive. Therefore, R is reflexive but neither symmetric nor transitive.
2.	Show that the relation R in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4,4), (1, 3), (3, 3), (3, 2)} is reflexive and transitive but not symmetric. **Solution:** Let A = {1, 2, 3, 4} The relation R is defined on A is given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)} Now, we show that the relation R is reflexive and transitive but not symmetric. <u>Reflexive</u>**:** Clearly, $(a, a) \in R\ for\ every\ a \in A$ Hence, R is reflexive. <u>Symmetric:</u> Clearly, $(1, 2) \in R,\ \ but\ (2, 1) \notin R$ Thus, for every $(a, b) \in R,\ \ (b, a) \notin R$ Hence, R is **not** symmetric, <u>Transitive:</u> For every $(a, b) \in R\ and\ (b, c) \in R\ \Rightarrow\ (a, c) \in R$ Hence, R is transitive. Therefore, R is reflexive and transitive but not symmetric.
3.	Check whether the relation R defined in the set A= $\{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b): b = a + 1, a, b \in A\}$ is reflexive, symmetric or transitive. **Solution:** Let A = {1, 2, 3, 4, 5, 6} R be the relation defined as $R = \{(a, b): b = a + 1\}$ i.e; R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)} **<u>To check R is reflexive:</u>** Clearly, $(a, a) \notin R\ for\ every\ a \in A$ Hence, R is **not** reflexive **<u>To check R is symmetric:</u>** Clearly, $(1, 2) \in R,\ \ but\ (2, 1) \notin R$

	Thus, for every $(a, b) \in R,\ \ (b, a) \notin R$ Hence, R is **not** symmetric **<u>To check R is Transitive:</u>** Take $(1,2) \in R,\ and\ (2,3)\ \in R\ but\ (1,3) \notin R$ Thus, $(a, b) \in R\ and\ (b, c) \in R\ then\ (a, c)\ ) \notin R$ Hence, R is not transitive. Therefore, R is neither reflexive, nor symmetric and nor transitive.
4.	Determine whether the relation R in the set $A\ =\ \{1, 2, 3, \dots ,13, 14\}$ defined as $R\ =\ \{(x, y) \colon 3x - y\ =\ 0, x, y \in A\}$ is reflexive or symmetric or transitive. **Solution:** Given $A\ =\ \{1, 2, 3, \dots ,13, 14\}$ The relation R is defined as R = {(1, 3), (2, 6), (3, 9), (4, 12)} **<u>To check R is reflexive:</u>** Clearly, $(a, a) \notin R\ for\ every\ a \in A$ Hence, R is **not** reflexive **<u>To check R is symmetric:</u>** Clearly, $(1, 3) \in R,\ \ but\ (3, 1) \notin R$ Thus, For every $(a, b) \in R,\ \ (b, a) \notin R$ Hence, R is **not** symmetric **<u>To check R is Transitive:</u>** Take $(1,3) \in R,\ and\ (3,9)\ \in R\ but\ (1,9) \notin R$ Thus, $(a, b) \in R\ and\ (b, c) \in R\ then\ (a, c)\ ) \notin R$ Hence, R is not transitive. Therefore, R is neither reflexive, nor symmetric and nor transitive.
5.	Determine whether the relation R in the set N of natural numbers defined as $R\ =\ \{(x, y) \colon y\ =\ x\ +\ 5\ and\ x\ <\ 4\}$ is reflexive, symmetric or transitive. **Solution:** Given N = Set of all natural numbers. The relation R is defined on the set N as $R\ =\ \{(x, y) \colon y\ =\ x\ +\ 5\ and\ x\ <\ 4\}$ i.e; R = {(1, 6), (2, 7), (3, 8)} **<u>To check R is reflexive:</u>** Clearly, $(a, a) \notin R\ for\ every\ a \in N$ Hence, R is **not** reflexive **<u>To check R is symmetric:</u>** Clearly, $(1, 6) \in R,\ \ but\ (6, 1) \notin R$ Thus, For every $(a, b) \in R,\ \ (b, a) \notin R$ Hence, R is **not** symmetric **<u>To check R is Transitive:</u>** For transitive, $we\ have\ to\ show\ for\ \ (a, b) \in R\ and\ (b, c) \in R\ then\ (a, c) \in\ R$ But in this case, in the relation R, for every order pair (a, b), there exists no order pair as (b, c). In such a case, R is also transitive. Therefore, the relation R is neither reflexive nor symmetric but transitive.
6.	Prove that the Greatest Integer Function $f\ \colon\ R\ \to\ R$, given by $f(x)\ =\ [x]$, is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x. **Solution:** f : R → R, given by f(x) = [x]

<table>
<tr><td></td><td>
Now, we prove that f is neither one-one nor onto
<u>To Prove f is not one-one:</u>
$Clearly, we\ have\ that\ f(1.1) = [1.1] = 1$
$f(1.2) = [1.2] = 1, \ldots\ldots\ldots\ldots\ldots, f(1.9) = [1.9] = 1$
From this, we conclude that different elements in the domain of f have same images in the co-domain of f.
Hence, f is not one-one function.
<u>To Prove f is not onto:</u>
$we\ know\ that\ codomain\ of\ f = R\ \ (set\ of\ all\ real\ numbers)$
$Range\ of\ f = Z\ \ (set\ of\ all\ integers)$
$Clearly, codomain\ of\ f\ \neq Range\ of\ f$
Hence, f is not onto.
Therefore, f is neither one-one nor onto.
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<tr><td>7.</td><td>
Show that the Modulus Function f : R → R, given by $f(x) = |\ x\ |$, is neither one-one nor onto, where $|\ x\ |is\ x$, if x is positive or 0 and $|\ x\ |is - x, if\ x$ is negative.
Solution: $f: R \longrightarrow R\ is\ given\ by f(x) = |x| = \begin{cases} x, if\ x \geq 0 \\ -x, if\ x < 0 \end{cases}$
Now, we prove that f is neither one-one nor onto
<u>To Prove f is not one-one:</u>
$Clearly, we\ have\ that\ f(1) = 1 = f(-1)$
$f(2) = 2 = f(-2)\ and\ so\ on.$
From this, we conclude that different elements in the domain of f have same images in the co-domain of f.
Hence, f is not one-one function.
<u>To Prove f is not onto:</u>
$we\ know\ that\ codomain\ of\ f = R\ \ (set\ of\ all\ real\ numbers)$
$Range\ of\ f = R^+\ \ (set\ of\ all\ non-negative\ real\ numbers)$
$Clearly, codomain\ of\ f\ \neq Range\ of\ f$
Hence, f is not onto.
</td></tr>
<tr><td>8.</td><td>
Show that the function $f: R \longrightarrow R\ is\ given\ by f(x) = \begin{cases} 1, if\ x > 0 \\ 0,\ if\ x = 0 \\ -1, if\ x < 0 \end{cases}$ (this function is called signum function) is neither one-one nor onto.
Solution: The function $f: R \longrightarrow R\ is\ given\ by f(x) = \begin{cases} 1, if\ x > 0 \\ 0,\ if\ x = 0 \\ -1, if\ x < 0 \end{cases}$
Now, we prove that f is neither one-one nor onto.
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<u>To Prove f is not one-one:</u>
$Clearly, we\ have\ that\ f(1) = 1, \quad f(2) = 1, f(3) = 1, and\ so\ on.$
$f(-1) = -1,\ f(-2) = -2,\ and\ so\ on.$
From this, we conclude that different elements in the domain of f have same images in the co-domain of f.
Hence, f is not one-one function.
<u>To Prove f is not onto:</u>
$we\ know\ that\ codomain\ of\ f = R\ \ (set\ of\ all\ real\ numbers)$
$Range\ of\ f = \{-1, 0, 1\}$
$Clearly, codomain\ of\ f\ \neq Range\ of\ f$
Hence, f is not onto.
Therefore, f is neither one-one nor onto.
</td></tr>
<tr><td>9.</td><td>
$Prove\ that\ the\ function\ f: N \longrightarrow N\ defined\ by\ f(x) = x^2 + x + 1$ is one-one but not onto.
Solution: $The\ function\ \boldsymbol{f}: \boldsymbol{N} \longrightarrow \boldsymbol{N}\ \ given\ by\ \ f(x) = x^2 + x + 1$
Now we prove that f is one-one but not onto.
<u>To prove f is one-one:</u>
Let $x_1, x_2 \in N\ such\ that\ \ f(x_1) = f(x_2)$
$\Rightarrow x_1{}^2 + x_1 + 1 = x_2{}^2 + x_2 + 1$
$\Rightarrow x_1{}^2 - x_2{}^2 + x_1 - x_2 = 0$
$\Rightarrow (x_1{}^2 - x_2{}^2) + (x_1 - x_2) = 0$
$\Rightarrow (x_1 + x_2)(x_1 - x_2) + (x_1 - x_2) = 0$
$\Rightarrow (x_1 - x_2)[(x_1 + x_2) + 1] = 0$
$\Rightarrow (x_1 - x_2) = 0 \quad (\because [(x_1 + x_2) + 1] > 0\ as x_1, x_2\ in\ domain\ \boldsymbol{N})$
$\Rightarrow x_1 = x_2$
Hence, f is one-one
<u>To prove f is onto:</u>
we have $f(1) = 3, f(2) = 7$ and so on.
Thus, $f(x) = x^2 + x + 1 \ \geq 3\ for\ every\ x \in N\ (domain)$
Clearly, $f(x) not\ taking\ values\ 1\ and\ 2$
Thus, the every element of co-domain in N has no pre-image in the domain N
Hence, f is not onto.
</td></tr>
<tr><td>10.</td><td>
$Let\ f: W \rightarrow W be\ defined\ as\ f(x) = x - 1, if\ x\ is\ odd\ and$
$f(x) = x + 1, if\ x\ is\ even.$ Show that f is both one-one and onto.
Solution:
$f: W \rightarrow W\ be\ defined\ as\ f(x) = x - 1, if\ x\ is\ odd$ and
</td></tr>
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	$f(x) = x + 1, if\ x\ is\ even.$ **<u>To Prove f is one-one:</u>** **<u>Case I:</u>** when $x_1\ and\ \ x_2$ are even number Now, consider $f(x_1) = f(x_2)$ $\Rightarrow x_1 + 1 = x_2 + 1$ $\Rightarrow x_1 = x_2$ Hence, f is one-one **<u>Case II:</u>** when $x_1\ and\ \ x_2$ are odd number Now, consider $f(x_1) = f(x_2)$ $\Rightarrow x_1 - 1 = x_2 - 1$ $\Rightarrow x_1 = x_2$ Hence, f is one-one. **<u>Case III:</u>** when $x_1\ is\ odd\ and\ \ x_2$ is even number Here, $x_1 \neq x_2$ Also, in this case $f(x_1)\ is\ even\ and\ f(x_2) is\ odd.$ Hence, $f(x_1) \neq f(x_2)$ Therefore, f is one-one. **<u>To Prove f is onto:</u>** For every even number 'y' in co-domain there exists odd number $y + 1$ in domain and for every odd number 'y' in co-domain there exists even number $y - 1$ in domain such that $f(x) = y$. Hence, f is onto Therefore, f is both one-one and onto.

EXERCISE

1.	Consider $f: R^+ \longrightarrow [4, \infty) given\ by\ f(x) = x^2 + 4$. Show that f is both one-one and onto.
2.	Let $\mathrm{A} = \mathrm{R} - \left\{\frac{2}{3}\right\}$ and $\mathrm{B} = \mathrm{R} - \left\{\frac{2}{3}\right\}$. If f: A→ B and $\mathrm{f(x)} = \frac{2x-1}{3x-2}$, then prove that the function f is one-one and onto.
3.	Show that the function $f: R \to R$ defined by $f(x) = \frac{x}{x^2+1}$,$\forall\ x \in R$ is neither one-one nor onto.
4.	Let R be the relation on set of all integer Z defined by $R = \{(a, b): \|a - b\| \leq 3\}$. Check whether R is an equivalence relation. Answer: R is reflexive, symmetic but not transitive

5 MARKS QUESTIONS

1.	Let R be the relation in the set Z of integers given by $R = \{(a,b): 2\ divides\ a - b\}$. Show that the relation R equivalence? Write the equivalence class [0]. **Solution :** Given R = {(a, b) : 2 divides a – b} For equivance relation we have to check : **(i) Reflexive:** If $(a - b)$ is divisible by 2 then, $\Rightarrow (a - a) = 0$ is also divisible by 2 $\Rightarrow (a, a) \in R$ Hence R is Reflexive ∀ a,b ∈ Z (ii) **Symmetric**: If $(a - b)$ is divisible by 2 then, $\Rightarrow (b - a) = -(a - b)$ is also divisible by 2 $\Rightarrow (a, b) \in R\ and\ (b, a) \in R$ Hence R is Symmetric ∀ a,b ∈ Z (iii) **Transitive**: If $(a - b) and\ (b - c)$ are divisible by 2 then, $\Rightarrow a - c = (a - b) + (b - c)$ is also divisible by 2 ⇒ (a,b) ∈ R, (b,c) ∈ R and (a,c) ∈ R. Hence R is Transitive ∀ a,b,c ∈ Z ⇒ As Relation R is satisfying**Reflexive , Symmetric** and **Transitive.** Hence R is an equivalence relation. Now equivalence class [0] $R = \{(a,b): 2\ divides\ (a - b)\} \Rightarrow (\boldsymbol{a} - \boldsymbol{b})\ is\ a\ multiple\ of\ 2.$ To find equivalence class 0**,** put b=0 So, **(**a–0) is a multiple of 2 **⇒** a is a multiple of 2 So, In the set Z of integers, all the multiple of 2 will come in equivalence class [0] Hence, equivalence class $[0] = \{2x : x \in Z\}$
2.	Show that the relation R defined by $(a,b)R\ (c,d) \Leftrightarrow a + d = b + c$ on the set × N is an equivalence relation. Also, find the equivalence classes [(2,3)] and 1,3)]. **Solution:** Given that, R be the relation in N × N defined by (a, b) R (c, d) if $a + d = b + c$ for (a, b), (c, d) in N × N. **Reflexive:** Let (a, b) R (a, b) ⇒ a + b = b + a which is true since addition is commutative on N. ⇒ R is reflexive. **Symmetric**: Let (a, b) R (c, d) ⇒ a + d = b + c ⇒ b + c = a + d ⇒ c + b = d + a ⇒ (c, d) R (a, b) ⇒ R is symmetric. **Transitive** ⇒ (a, b) R (e, f) for (a, b), (c, d),(e, f) in N × N. Let (a, b) R (c, d) and (c, d) R (e, f) ⇒ a + d = b + c and c + f = d + e ⇒ (a + d) – (d + e) = (b + c) – (c + f) ⇒ a - e = b - f ⇒ a + f = b + e ⇒ (a, b) R (e, f) ⇒ R is transitive.

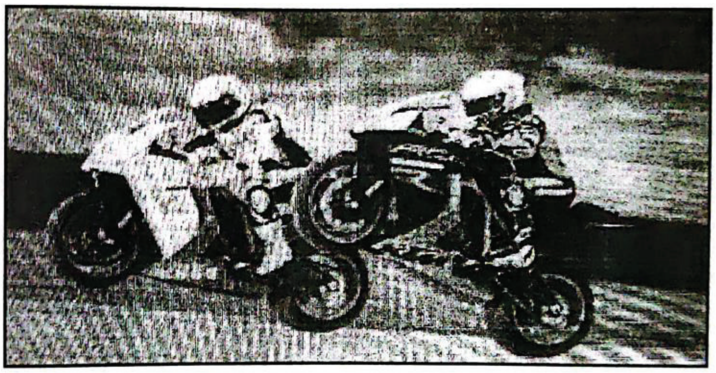
	Hence, R is an equivalence relation. **Equivalence Classes:** The equivalence class of (a, b) is the set of all pairs (c, d) such that a + d = b + c. $\Rightarrow a - b = c - d$ The equivalence classes of [(2,3)] is Put $a = 2$ *and* $b = 3$ c – d = 2 – 3 d – c = 1 [(2,3)] = {(c,d) : d – c = 1 ∀ c,d ∈ Z } The equivalence classes of [(1,3)] is Put a=1and b=3 c – d = 1 – 3 ⇒ d – c = 2 [(1,3)] = {(c,d) : d – c = 2 ∀c,d ∈ Z }
3.	Show that the relation R in the set of ℝ real numbers, defined as $R = \{(a,b): a \leq b^3\}$ is neither reflexive nor symmetric nor transitive. **Solution:** Given $\mathbb{R} = set\ of\ all\ real\ numbers$ The relation R in the set ℝ defined as $R = \{(a,b): a \leq b^3\}$ Now we prove that R is neither reflexive nor symmetric nor transitive. **<u>To show R is not reflexive:</u>** We know that $a \leq a^3$ is not true for any positive real number less than 1. For example, $for\ a = \frac{1}{2},\ \frac{1}{2} \nleq \left(\frac{1}{2}\right)^3 = \frac{1}{8}$ Thus, clearly, $(a,a) \notin R\ for\ every\ a \in \mathbb{R}$ Hence, R is **not** reflexive **<u>To show R is not symmetric:</u>** Take a = 1 and b = 2 Now, $a = 1 \leq 2^3 = b^3 \Rightarrow a \leq b^3 \Rightarrow (a,b) \in R$ But, $b = 2 \nleq (1)^3 = 1 = a \Rightarrow b \nleq a^2 \Rightarrow (b,a) \notin R$ Thus, $(a,b) \in R \Rightarrow (b,a) \notin R$ Hence, R is not symmetric. **<u>To show R is not transitive:</u>** Let us take a = 10, b = 4, c = 2 $(a,b) \in R = (10,4) \in R\ as\ 10 \leq 4^3 = 64$ $(b,c) \in R = (4,2) \in R\ as\ 4 \leq 2^3 = 8$ But, $(a,c) \notin R\ as\ 10 \nleq 2^3 = 8$ Thus, $(a,b) \in R\ and\ (b,c) \in R \Rightarrow (a,c) \notin R$ Hence, R is not tranitive. Therefore, the relation R is neither reflexive, nor symmetric nor transitive.
4.	Let A = {1, 2, 3, ……., 9} and $(a,b)R\ (c,d) if$ $ad = bc\ for\ (a,b),(c,d)\ in$ A×A. Prove that R is an equivalence relation**.** **Solution:** Given A = {1, 2, 3, ……., 9} The relation R is defined as $(a,b)R\ (c,d) if\ ad = bc\ for\ (a,b),(c,d) in$ A × A Now, we prove that R is an equivalence relation. **<u>To show R is reflexive:</u>** Clearly, $ab = ba$ for every a, b ∈ A $\Rightarrow (a,b)R(a,b)\ for\ every\ (a,b) \in$ A × A Hence, R is reflexive. **<u>To show R is symmetric:</u>** Let (a, b) R (c, d)

	$\Rightarrow ad = bc$ $\Rightarrow da = cb$ $\Rightarrow cb = da$ $\Rightarrow$ (c, d)R (a, b) Hence, R is symmetric **To show R is transitive:** Let (a, b) R (c, d) $\Rightarrow ad = bc$ …….. (1) And (c, d) R (e, f) $\Rightarrow cf = de$ ………. (2) Multiply (1) and (2), we get $(ad)\,(cf) = (bc)(de)$ $\Rightarrow af = be$ $\Rightarrow (a, b)R(e, f)$ Hence, R is transitive. Therefore, R is reflexive, symmetric and transitive. Hence, R is an equivalence relation.
5.	Consider a function $f\colon \left[0, \frac{\pi}{2}\right] \longrightarrow R\ given\ by\ f(x) = sinx\ and\ g\colon \left[0, \frac{\pi}{2}\right] \longrightarrow R$ given by $g(x) = cosx$. Show that $f\ and\ g$ are one-one but $f + g$ is not one-one. **Solution:** To prove f is one-one: $The\ function\ f\colon \left[0, \frac{\pi}{2}\right] \longrightarrow R\ given\ by\ f(x) = sinx$ Clearly, different elements in the domain $\left[0, \frac{\pi}{2}\right]$ of 'f' have distinct images in the co-domain of 'f' Hence, f is one-one. To prove g is one-one: $The\ function\ g\colon \left[0, \frac{\pi}{2}\right] \longrightarrow R\ given\ by\ g(x) = cosx$ Clearly, different elements in the domain $\left[0, \frac{\pi}{2}\right]$ of 'g' have distinct images in the co-domain of 'g' Hence, 'g' is one-one. To prove $f + g$ is one-one: $(f + g)(x) = f(x) + g(x) = sinx + cosx$ $(f + g)(0) = sin0 + cos0 = 0 + 1 = 1$ $(f + g)\left(\frac{\pi}{2}\right) = sin\frac{\pi}{2} + cos\frac{\pi}{2} = 1 + 0 = 1$ From this, we conclude that different elements in the domain of $f + g$ have same images in the co-domain of $f + g$. Hence, $f + g$ is not one-one.

EXERCISE

1	Let f : R → R be a function defined as $f(x) = 4x + 3$, then show that f is one-one and onto.
2	A function $f: [-4,4] \rightarrow [0,4]$ is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of ' a ' for which $f(a) = \sqrt{7}$.
3	A relation R is defined on a set of real numbers ℝ as $$R = \{(x, y): \ xy \ \text{ is an irrational number}\}$$ Check whether R is reflexive, symmetric and transitive or not.
4	Show that the relation R in the set A = $\{x \in Z: 0 \leq x \leq 12\}$, given by R = {(a, b): |a – b| is divisible by 3} is an equivalence relation.

CASE STUDY QUESTIONS

I	Read the passage given below and answer the questions that follow: An organisation conducted a bike race under two different categories boys and girls .Among all the participants finally three from category 1(boys) and two from category 2(girls) were selected for the final race. Let B=$\{b_1,b_2,b_3\}$,G=$\{g_1,g_2\}$, where B represents the set of boys selected and G the set of girls who were selected for the final race.  1. How many relations are possible from B to G?(1mark) 2. How many functions can be formed from B to G? (1mark) 3. How many one-one functions be formed from B to G? justify your answer. (2marks) **Solution:** **(1)** n(B)= 3, n(G)=2 Number of relations from B to G is 2^6 ,because every relation is a subset of B×G and there are 3×2=6 elements in B×G. **(2)**Number of functions= 2^3=8. **(3)** Number of one-one functions = 0 as n(B) > n(G).

II Priya and Surya are playing monopoly in their house during COVID. While rolling the dice their mother Chandrika noted the possible outcomes of the throw every time belongs to the set $\{1, 2, 3, 4, 5, 6\}$. Let A denote the set of players and B be the set of all possible outcomes. Then $A = \{P, S\}$ $\quad B = \{1, 2, 3, 4, 5, 6\}$.Then answer the below questions based on the given information.(each question carries one mark)

(a) Let $R: B \rightarrow B$ be defined by $R = \{(a, b)\ both\ a\ and\ b\ are\ either\ odd\ or\ even\}$, then is R an Equivalence relation?

(b) Chandrika wants to know the number of functions from A to B. How many numbers of functions are possible?

(c) Let R be a relation on B defined by $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$. Then is R relexive, symmetric and transitive?

(d) Let $R: B \rightarrow B$ be defined by $R = \{(1, 1), (1, 2), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$ then is R symmetric? Justify

Solution:

(a) Yes, it is equivalence.
The relation is reflexive,symmetric and transitive hence it is Equivalence.

(b) If $n(A) = m, n(B) = n$, then the number of functions from A to B is n^m
Ans: 6^2

(c) As $(1,1) \notin R$ It is not reflexive
As $(1,2) \in R$ but $(2,1) \notin R$ it is not symmetric
As $(1,3) \in R$ and $(3, 4) \in R$ but $(1,4) \notin R$,it is not transitive

Hence none.

(d) No, $(1,2) \in R$ but $(2,1) \notin R$ it is not symmetric

III In two different societies, there are some school going students – including girls as well as boys. Satish forms two sets with these students, as his college project.

Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ and $B = \{b_1, b_2, b_3, b_4\}$ where $a_i's, b_i's$are the school going students of first and second society respectively.

Using the information given above, answer the following question

(a) Satish wishes to know the number of reflexive relations defined on set A. How many such relations are possible? (1mark)

(b) Let $R: A \to A, R = \{(x, y): x \text{ and } y \text{ are students of same sex}\}$. Then relation is R an equivalence relation. (1 mark)

(c) Let $R: A \to B, R = \{(a_1, b_1), (a_2, b_1), (a_3, b_3), (a_4, b_2), (a_5, b_2)\}$, then is R One-one and onto ? Justify (2marks)

Solution:

(a) 2^{20}

If $n(A) = n$ then number of reflexive relations that can be defined on A is 2^{n^2-n}.here $n(A) = 5$.

(b) Yes the relation is equivalence.

As the relation is reflexive,symmetric and transitive.

(c) No R is not one-one and onto

Distinct elements of A have same image a_1 *and* a_2 are having same

Image b_1 ,so R is not one-one.

b_4 is not having pre-image ,so R is not onto.

EXERCISE

I

Vani and Mani are playing Ludo at home while it was raining outside.While rolling the dice Vani's brother Varun observed and noted the possible outcomes of the throw every time belongs to the set {1,2,3,4,5,6}.Let A be the set of players while B be the set of all possible outcomes.

A= {Vani, Mani}, B= {1, 2, 3, 4, 5, 6}.

Answer the following questions:

a) Let $R: B \to B$,be defined by $R = \{(x, y): y \text{ is divisible by } x\}$.Verify that whether R is reflexive, symmetric and transitive.(2marks)

b) Is it possible to define an onto function from A to B? Justify.(1mark)

c) Which kind of relation is R defined on B given by

R={(1,2),(2,2),(1,3),(3,4),(3,1),(4,3),(5,5)}?(1mark)

Or

Find the number of possible relations from A to B.

Answer:a) R is reflexive and transitive but not symmetric.

b) No. Because n(B) is greater than n(A)

c) R is neither reflexive nor symmetric nor transitive

or

no. of relations= 2^{12}

II	 Students of class X planned to plant saplings along straight lines,parallel to each other to one side of the playground ensuring that thy had enough play area.Let us assume that they planted one of the rows of the saplings along the line $y = x + 4$. Let L be the set of all lines which are parallel on the ground and R be a relation on L. i) R = $\{(l_1, l_2): l_1 \perp l_2, where\ l_1, l_2 \in L\}$. What is the type of relation R? (1mark) ii) R = $\{(l_1, l_2): l_1 \|\| l_2,\ where\ \ l_1, l_2 \in L\}$. What is the type of relation R? (1mark) iii) Check whether the function $f: R \rightarrow R\ defined\ by\ f(x) = x + 4$ is one-one and onto. (2marks) **Answer:** i) R is symmetric but neither reflexive nor transitive. ii) R is an equivalence relation. iii)R is both one-one and onto