INVERSE TRIGONOMETRIC FUNCTIONS

MULTIPLE CHOICE QUESTIONS

Q.NO	QUESTIONS AND SOLUTIONS
1	The principal value of $cos^{-1}\left(\frac{-1}{2}\right)$`  (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (iii) $\frac{-\pi}{3}$ (d) $\frac{-\pi}{6}$  **Solution:** We have $cos^{-1}(-x) = \pi - cos^{-1}(x)$  $cos^{-1}\left(\frac{-1}{2}\right)$`$= \pi - cos^{-1}\left(\frac{1}{2}\right)$. $= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ Ans: (a)
2	The principal value of $sin^{-1}\left[sin\left(\frac{3\pi}{5}\right)\right]$` (a) $\frac{3\pi}{5}$ (b) $\frac{2\pi}{5}$ (iii) $\frac{-2\pi}{5}$ (d) $\frac{\pi}{5}$ **Solution:** We have $sin^{-1}\left[sin\left(\frac{3\pi}{5}\right)\right] = sin^{-1}\left[sin\left(\pi - \frac{3\pi}{5}\right)\right]$ $= sin^{-1}\left[sin\left(\frac{2\pi}{5}\right)\right]$ $= \frac{2\pi}{5}$ Ans. (b)
3	The value of: $tan^{-1}\sqrt{3} - sec^{-1}(-2)$ is (a) $\frac{\pi}{6}$ (b) $\frac{-\pi}{6}$ (c) $\frac{-\pi}{3}$ (d) 0 **Solution:** We have $sec^{-1}(-x) = \pi - sec^{-1}(x)$. $\therefore \tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - (\pi - \frac{\pi}{3}) = \frac{-\pi}{3}$ **Ans: (c)**
4	The value of $sin\left[\frac{\pi}{3} - sin^{-1}\left(\frac{-1}{2}\right)\right]$ is (a) 0 (b) 1 (c) -1 (d) 2 **Solution:** We have $\sin^{-1}(-x) = -\sin^{-1}(x)$

	$\therefore \sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right] = \sin\left[\frac{\pi}{3} - \left(\frac{-\pi}{6}\right)\right] = \sin\left(\frac{\pi}{2}\right) = 1$ Ans: (b)
5	The principal value of $cos^{-1}\left(cos\left(\frac{-7\pi}{3}\right)\right)$ is (a) $\frac{7\pi}{3}$ (b) $\frac{\pi}{3}$ (iii) $\frac{-\pi}{3}$ (d) $\frac{-7\pi}{3}$ **Solution:** We have $\cos(-x) = cosx$ $\therefore \cos^{-1}\cos\left(\frac{-7\pi}{3}\right) = \cos^{-1}\cos\left(\frac{7\pi}{3}\right)$ $= \cos^{-1}\cos\left(2\pi + \frac{\pi}{3}\right)$ $= \cos^{-1}\cos\left(\frac{\pi}{3}\right) = \frac{\pi}{3}$ Ans: (b)
6	The value of $tan^{-1}\sqrt{3} - cot^{-1}(-\sqrt{3})$ is (a) $\frac{\pi}{2}$ (b) $\frac{-\pi}{2}$ (c) $\frac{-\pi}{3}$ (d) $\frac{\pi}{6}$ **Solution:** $\cot^{-1}(-\sqrt{3}) = \pi - \cot^{-1}(\sqrt{3}) = \pi - \frac{\pi}{6} = 5\frac{\pi}{6}$ $\therefore \tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3}) = \frac{\pi}{3} - \frac{5\pi}{6} = -\frac{\pi}{2}$ **Ans: (b)**
7	The value of x if $tan^{-1}\sqrt{3} + cot^{-1}x = \frac{\pi}{2}$ (a) $\sqrt{3}$ (b) $-\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\pi}{6}$ **Solution:** $\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}\sqrt{3} = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$ $\therefore x = \cot\frac{\pi}{6} = \sqrt{3}$ **Ans: (a)**

8	The value of x if $sec^{-1}2 + cosec^{-1}x = \frac{\pi}{2}$ (a) $\sqrt{3}$ (b) 2 (c) $\frac{\sqrt{3}}{2}$ (d) -2 **Solution:** $\cos ec^{-1}x = \frac{\pi}{2} - \sec^{-1}\sqrt{3}$ = $\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$ $\therefore x = \cos ec\frac{\pi}{6} = 2$ **Ans: (b)**
9	If $sin^{-1}x = y$ then the principal value of y is: (a) $0 \le y \le \pi$ (b) $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$ c) $\frac{-\pi}{2} < y < \frac{\pi}{2}$ (d) $0 < y < \pi$ **Ans: (b)**
10	If $tan^{-1}x = y$ then the principal value of y is: (a) $0 \le y \le \pi$ (b) $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$ c) $\frac{-\pi}{2} < y < \frac{\pi}{2}$ (d) $0 < y < \pi$ **Ans: (c)**

EXERCISE

1	The value of $cos\left(tan^{-1}\frac{3}{4}\right)$ is : (a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{3}{7}$

	Answer: (b) $\frac{4}{5}$
2	The principal value of : $tan^{-1}\left[tan\left(\frac{5\pi}{4}\right)\right]$ (a) $\frac{5\pi}{4}$ (b) $\frac{\pi}{4}$ (iii) $\frac{-\pi}{4}$ (d) 1 Answer: (b) $\frac{\pi}{4}$
3	The value of $cot\ \left(cos^{-1}\frac{7}{25}\right)$ is : (a) $\frac{7}{24}$ (b) $\frac{24}{25}$ (c) $\frac{7}{25}$ (d) $\frac{25}{7}$ Answer: (a) $\frac{7}{24}$
4	The value of $cos^{-1}\left(cos\left(\frac{14\pi}{3}\right)\right)$ is : (a) $\frac{14\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{4\pi}{3}$ Answer: (c) $\frac{2\pi}{3}$
5	The value of $2\,sin^{-1}\left(\frac{1}{2}\right) + cot^{-1}(1)$ (a) $\frac{7\pi}{12}$ (b) $\frac{3\pi}{4}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{4}$ Answer:(a) $\frac{7\pi}{12}$

ASSERTION-REASON BASED QUESTIONS

	In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.
1	ASSERTION (A): Principal value of $cos^{-1}\,cos\left(\frac{7\pi}{6}\right)$ is $\frac{5\pi}{6}$ REASON (R): Range of principal branch of $\cos^{-1}$ is $[\,0, \pi]$ and $\cos^{-1}(\cos x) = x$ if $x \in [\,0, \pi]$. Ans: (a)
2	ASSERTION (A): Principal value of $sin^{-1}\,sin\left(\frac{13\pi}{6}\right)$ is $\frac{\pi}{6}$ REASON (R): $\sin^{-1}(-x) = -\sin^{-1}(x)$ Ans:(b)
3	ASSERTION (A): Principal value of $sin^{-1}(-1) = \frac{-\pi}{2}$ REASON (R): $\sin^{-1}(-x) = -\sin^{-1}(x)$ Ans: (a)

4	ASSERTION (A): Principal value of $sin^{-1}\, sin\left(\frac{3\pi}{5}\right) = \frac{3\pi}{5}$ REASON (R): $\sin^{-1}\sin(x) = x, x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ Ans: (d)
5	ASSERTION (A): The principal value of $cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$, REASON (R): cosine function is an even function, therefore $\cos(-x) = cosx$. Ans: (d)
6	ASSERTION (A): The principal value of $cos^{-1}\left(\frac{-1}{2}\right) = \pi - cos^{-1}\left(\frac{1}{2}\right)$, REASON (R): Range of $\cos^{-1}$x is $[0, \pi]$ Ans: (b)
7	ASSERTION (A): The principal value of $tan^{-1}\left[sin\left(\frac{-\pi}{2}\right)\right] = \frac{-\pi}{2}$, REASON (R): $\tan^{-1}(-x) = \tan^{-1}(x)$ Ans: (d)
8	ASSERTION (A): The principal value of $tan^{-1}\, tan\left(\frac{-\pi}{4}\right) = \frac{-\pi}{4}$, REASON (R): Range of $\tan^{-1}$ x is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, $\tan^{-1}(\tan x) = x$ if $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ Ans: (a)
9	ASSERTION(A): One branch of $cos^{-1}x$ other than the principal value branch is $[\pi, 2\pi]$ REASON (R): $cos\left(\frac{-\pi}{2}\right) = -1$ Ans: (c)
10	ASSERTION (A): One branch of $sin^{-1}x$ other than the principal value branch is $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ REASON (R): $sin\left(\frac{3\pi}{2}\right) = sin\left(\frac{\pi}{2}\right) = 1$ Ans: (c)

EXERCISE

1	ASSERTION (A): $sin(cos^{-1}x) = cos(sin^{-1}x) = \sqrt{1-x^2}, \quad |x| \leq 1$ REASON (R): Because $sin^2\theta + cos^2\theta = 1$ Answer: (b)
2	ASSERTION (A): The principal value of $cos^{-1}\left[cos\left(\frac{-\pi}{4}\right)\right] = \frac{-\pi}{4}$ REASON (R): Range of $\cos^{-1}x$ is $[0, \pi]$ $\cos^{-1}(cosx) = x$ if $x \in [0, \pi]$ Answer: (d)
3	ASSERTION (A): The principal value of $sin[cot^{-1}(cos(tan^{-1}1))] = \sqrt{\frac{2}{3}}$

	REASON (R): Range of $\tan^{-1}$ x is $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$, $\tan^{-1}(tanx) = x$ if $x\epsilon$ $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ Answer: (b)
4	ASSERTION (A): The principal value of $cos^{-1}\,cos\left(\frac{-\pi}{4}\right) = \frac{\pi}{4}$ REASON (R): Cosine function is an even function, therefore $\cos(-x) = cosx$. and $\cos^{-1}(cosx) = x$ if $x\epsilon\,[0,\pi]$ Answer: (a)
5	ASSERTION (A): The value of $sin\left[2\,sin^{-1}\left(\frac{3}{4}\right)\right] = \frac{3}{4}$ REASON (R): $sin(\,sin^{-1}x) = x,\quad x \in [-1,,1]$ Answer: (d)

2 MARK QUESTIONS

1	Find the value of $sin^{-1}\left[sin\left(\frac{2\pi}{3}\right)\right] + cos^{-1}\left[cos\left(\frac{2\pi}{3}\right)\right]$. **Solution:** We have $sin^{-1}\,sin\left(\frac{2\pi}{3}\right) = sin^{-1}\,sin\left(\pi - \frac{\pi}{3}\right)$ $= sin^{-1}\,sin\left(\frac{\pi}{3}\right)$ $= \frac{\pi}{3}$ $Value\quad of\quad cos^{-1}\,cos\left(\frac{2\pi}{3}\right) = \frac{2\pi}{3}$ $\therefore sin^{-1}\,sin\left(\frac{2\pi}{3}\right) + cos^{-1}\,cos\left(\frac{2\pi}{3}\right) = \frac{\pi}{3} + \frac{2\pi}{3} = \pi$
2	Find the value of: $tan^{-1}\left[2\,cos(\,sin^{-1}\left(\frac{1}{2}\right))\right]$ **Solution:** $tan^{-1}\left[2\,cos(\,sin^{-1}\left(\frac{1}{2}\right))\right] = tan^{-1}\left[2\,cos\left(\frac{\pi}{6}\right)\right] = tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right] = tan^{-1}\sqrt{3} = \frac{\pi}{3}$
3	Find the value of: $tan^{-1}\left[2\,sin(\,2\,cos^{-1}\left(\frac{\sqrt{3}}{2}\right))\right]$ **Solution:** $tan^{-1}\left[2\,sin(\,2\,cos^{-1}\left(\frac{\sqrt{3}}{2}\right))\right] = tan^{-1}\left[2\,sin(\,2\times\frac{\pi}{6})\right]$ $= tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right] = tan^{-1}\sqrt{3} = \frac{\pi}{3}$
4	If $cot^{-1}\left(\frac{1}{5}\right) = x, then\ find\ the\ value\ of\ \sin x$,

	Solution: $cotx = \frac{1}{5}$ $\therefore \sin x = \frac{5}{\sqrt{26}}$
5	Find the value of $sin^{-1}\left(\frac{-1}{2}\right) + 2\,cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ **Solution:** $sin^{-1}\left(\frac{-1}{2}\right) = -sin^{-1}\left(\frac{1}{2}\right) = -\frac{\pi}{6}$ $cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \pi - cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ $\therefore sin^{-1}\left(\frac{-1}{2}\right) + 2\,cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{-\pi}{6} + 2 \times \frac{5\pi}{6} = \frac{3\pi}{2}$
6	Show that for $\lvert x\rvert < 1,\ sin(tan^{-1} x) = \frac{x}{\sqrt{1+x^2}}$ **Solution:** Let $tan^{-1} x = y$ $\therefore \quad tany = x$ $\therefore$ L H S = $\sin y = \frac{x}{\sqrt{1+x^2}} \quad = R\ H\ S$
7	Prove that: $tan\left(\frac{1}{2} sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$ **Solution**: Let $sin^{-1}\frac{3}{4} = x$ $\therefore \sin x = \frac{3}{4}$ $\therefore \cos x = \frac{\sqrt{7}}{4}$ L H S = $tan\frac{x}{2} = \sqrt{\frac{1-cos x}{1+cos x}} = \sqrt{\frac{1-\frac{\sqrt{7}}{4}}{1+\frac{\sqrt{7}}{4}}} = \frac{4-\sqrt{7}}{3} = RHS$
8	Find the value of $tan\left(2\,tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$ **Solution:** Let $tan^{-1}\frac{1}{5} = x$ $\therefore \tan x = \frac{1}{5}$ $\therefore \quad tan\left(2\,tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right) = tan\left(2x - \frac{\pi}{4}\right) = \frac{tan\,2x - tan\frac{\pi}{4}}{1+tan\,2x\,tan\frac{\pi}{4}} = \frac{-7}{17}$ where, $tan\,2\,x = \frac{2\,tan\,x}{1-tan^2 x} = \frac{2\times\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2} = \frac{5}{12}$

9	Find the value of $sin^{-1}\left(\frac{-1}{2}\right) + 2\,cos^{-1}\left(\frac{-1}{2}\right) + tan^{-1}(1)$ **Solution:** $sin^{-1}\left(\frac{-1}{2}\right) = -sin^{-1}\left(\frac{1}{2}\right) = -\frac{\pi}{6}$ $cos^{-1}\left(\frac{-1}{2}\right) = \pi - cos^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ $tan^{-1}(1) == \frac{\pi}{4}$ $\therefore sin^{-1}\left(\frac{-1}{2}\right) + 2\,cos^{-1}\left(\frac{-1}{2}\right) + tan^{-1}(1) = \frac{-\pi}{6} + \frac{2\pi}{3} + \frac{\pi}{4} = \frac{3\pi}{4}$
10	Find the value of $sin\left(2\,sin^{-1}\frac{3}{5}\right)$ **Solution:** Let $sin^{-1}\left(\frac{3}{5}\right) = \theta$ $\therefore\ sin\,\theta = \frac{3}{5}$ $\therefore\ sin\left(2\,sin^{-1}\frac{3}{5}\right) = sin\,2\,\theta = 2\,sin\,\theta\,cos\,\theta$ $= 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$

2 MARKS

1	Find the value of $\sin\left[2\cot^{-1}\left(-\frac{5}{12}\right)\right]$ Answer: $-\frac{120}{169}$
2	Find the value of $tan\left[\frac{\pi}{6} - tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$ Answer: 0
3	Find the value of $sin\ \ (cot^{-1}\,x)$ in terms of x Answer: $\frac{1}{\sqrt{1+x^2}}$
4	Find the value of $sin\left[cot^{-1}\left(\frac{4}{3}\right)\right]$ Answer: $\frac{3}{5}$
5	Find the principal of $tan^{-1}\,tan\left(\frac{7\pi}{6}\right) + cot^{-1}\,cot\left(\frac{7\pi}{6}\right)$ Answer: $\frac{\pi}{3}$