

CH. 1. RELATIONS & FUNCTIONS

CLASS XII

MATHEMATICS

Time : 1½ hrs

Marks : 40

SET A

SECTION - A

10 × 1 = 10

- Let $A = \{1, 2, 3, 4\}$. Let R be the equivalence relation on $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$. Then the equivalence class $[(1, 3)]$ is
 a) $\{(1, 3)\}$ b) $\{(2, 4)\}$ c) $\{(1, 8), (2, 4), (1, 4)\}$ d) $\{(1, 3), (2, 4)\}$
- The maximum number of equivalence relations on the set $A = \{2, 3, 4\}$ are
 a) 1 b) 27 c) 3 d) 5
- Which of the following functions form \mathbb{Z} into \mathbb{Z} bijections?
 a) $f(x) = x^3$ b) $f(x) = x + 2$ c) $f(x) = 2x + 1$ d) $f(x) = x^2 + 1$
- If $R = \{(a, a^3) : a \text{ is a prime number less than } 5\}$ be a relation. Find the range of R .
 a) $\{8, 27\}$ b) $\{4, 25\}$ c) $\{8, 30\}$ d) $\{9, 25\}$
- Let R be a relation on the set L of lines defined by $l_1 R l_2$ if l_1 is perpendicular to l_2 , then relation R is
 a) reflexive and symmetric b) symmetric and transitive
 c) equivalence relation d) symmetric
- Let R is the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$. Write the equivalence class $[0]$.
 a) $\{3, 5\}$ b) $\{2, 4\}$ c) $\{4, 5\}$ d) $\{6, 6\}$
- Set A has 3 elements and the set B has 4 elements. Then the number of injective functions that can be defined from set A to set B is
 a) 144 b) 12 c) 24 d) 64
- If $R = \{(x, y) : x + 2y = 8\}$ is a relation on \mathbb{N} , then write the range of R .
 a) $\{4, 4, 1\}$ b) $\{3, 2, 1\}$ c) $\{6, 6, 2\}$ d) $\{7, 8, 2\}$

For question number 9 -10 two statements are given - one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

- Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
- Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

- c) Assertion (A) is true but Reason (R) is false.
 d) Assertion (A) is false but Reason (R) is true.
9. Assertion (A) : The relation R on the set of integers Z given by $R = \{(a, b) : a > b\}$ is an equivalence relation.
- Reason (R) : Any relation R on a set A is an equivalence relation, if it is reflexive, symmetric and transitive.
10. Assertion (A) : Let L be the collection of all lines in a plane and R_1 be the relation on L as $R_1 = \{(L_1, L_2) : L_1 \parallel L_2\}$ is symmetric relation.
- Reason (R) : Any relation R on a set A is symmetric relation, if $(a_1, a_2) \in R$ implies that $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

SECTION - B**2 × 2 = 4**

11. Let $f : A \rightarrow R$ be the function defined by $f(x) = \frac{1}{2 - \cos x}$, $\forall x \in R$. Then, find the range of f .
12. If functions $f : A \rightarrow B$ and $g : B \rightarrow A$ satisfy $gof = I_A$, then show that f is one-one and g is onto.

OR

Let n be a fixed positive integer. Define a relation R in Z as follows $\forall a, b \in Z$, aRb if and only if $a - b$ is divisible by n . Show that R is an equivalence relation.

SECTION - C**4 × 3 = 12**

13. Show that the relation R in the set of real numbers, defined as $R = \{(a, b) : \{(a, b) : a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.
14. Show that $f : N \rightarrow N$, given by $f(x) = \begin{cases} x + 1, & \text{if } x \text{ is odd} \\ x - 1, & \text{if } x \text{ is even} \end{cases}$ is bijective (both one-one and onto).
15. If $f : X \rightarrow Y$ is a function. Define a relation R on X given by $R = \{(a, b) : f(a) = f(b)\}$. Show that R is an equivalence relation on X .
16. If R is a relation defined on the set of natural numbers N as follows :
- $R = \{(x, y), x \in N, Y \in N \text{ and } 2x + y = 24\}$, then find the domain and range of the relation R . Also, find if R is an equivalence relation or not.

SECTION - D

2 × 5 = 10

17. Let N be the set of all natural numbers and let R be a relation on $N \times N$ defined by $(a, b) R (c, d) \Leftrightarrow ad = bc$ for all $(a, b), (c, d) \in N \times N$. Show that R is an equivalence relation on $N \times N$.

OR

If R_1 and R_2 are two equivalence relations in a set A , show that $R_1 \cap R_2$ is also an equivalence relation.

18. Show that the function $f : R \rightarrow \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in R$ is one-one and onto function.

SECTION - E

Case Study Based Questions

19. Read the following passage and answer the questions

In two different societies, there are some school going students - including girls as well as boys. Satish forms two sets with these students, as his college project.

Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ and $B = \{b_1, b_2, b_3, b_4\}$ where a_i 's, b_i 's are the school going students of first and second society respectively.

- i) Satish wishes to know the number of reflexive relations defined on set A . How many such relations are possible? 1
- ii) Satish and his friend Rajat are interested to know the number of symmetric relations defined on both the sets A and B , separately. Satish decides to find the symmetric relation on set A , while Rajat decides to find symmetric relation on set B . What is difference between their results? 2

OR

To help Satish in his project, Rajat decides to form onto function from set A to itself. How many such functions are possible?

- iii) Given set $A = \{a, b, c\}$. Write an identity relation in set A . 1

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SET B

SECTION - A

10 × 1 = 10

- If a relation R on the set $\{1, 2, 3\}$ be defined by $R = \{(1, 2)\}$, then R is
 a) reflexive b) transitive c) symmetric d) none of these
- If $f: R \rightarrow R$ be defined by $f(x) = \frac{2}{x}, x \in R$, then f is
 a) one-one b) onto c) bijective d) f is not defined
- If $A = \{5, 6, 7\}$ and let $R = \{(5, 5), (6, 6), (7, 7), (5, 6), (6, 5), (6, 7), (7, 6)\}$. Then R is
 a) Reflexive, symmetric but not Transitive
 b) Symmetric, transitive but not reflexive
 c) Reflexive, Transitive but not symmetric
 d) an equivalence relation
- The number of elements in set P is 4. The number of possible symmetric relations that can be defined on P is
 a) 16 b) 32 c) 512 d) 1024
- Let $P = \{a, b, c\}$. Then the number of Equivalence relations containing (a, b) is
 a) 1 b) 2 c) 3 d) 4
- Given set $A = \{a, b, c\}$. An identity relation in set A is
 a) $R = \{(a, b), (a, c)\}$ b) $R = \{(a, a), (b, b), (c, c)\}$
 c) $R = \{(a, a), (b, b), (c, c), (a, c)\}$ d) $R = \{(c, a), (b, a), (a, a)\}$
- A relation S in the set of real numbers is defined as $xSy \Leftrightarrow x - y + \sqrt{3}$ is an irrational number, then relation S is
 a) reflexive b) reflexive and symmetric
 c) transitive d) symmetric and transitive
- Range of the function $f(x) = \frac{|x-1|}{x-1}, x \neq 1$ is
 a) $\{-1, 1\}$ b) $\{1, 2\}$ c) $\{2, -1\}$ d) $\{1, -2\}$

For question number 9 -10 two statements are given - one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

- a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
 b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
 c) Assertion (A) is true but Reason (R) is false.
 d) Assertion (A) is false but Reason (R) is true.
9. Assertion (A) : If $n(A) = m$, then the number of reflexive relations on A is m .
 Reason (R) : A relation R on the set A is reflexive if $(a, a) \in R, \forall a \in A$.
10. Assertion (A) : The function $f : R \rightarrow R, f(x) = |x|$ is not one-one.
 Reason (R) : The function $f(x) = |x|$ is not onto.

SECTION - B

2 × 2 = 4

11. Let the function $f : R \rightarrow R$ be defined by $f(x) = \cos x, \forall x \in R$. Show that f is neither one-one nor onto.
12. Let $X = \{1, 2, 3\}$ and $Y = \{4, 5\}$. Find whether the following subsets of $X \times Y$ are functions from X to Y or not.
- i) $f = \{(1, 4), (1, 5), (2, 4), (3, 5)\}$ ii) $g = \{(1, 4), (2, 4), (3, 4)\}$
 iii) $h = \{(1, 4), (2, 5), (3, 5)\}$ iv) $k = \{(1, 4), (2, 5)\}$

OR

If $A = \{1, 2, 3\}$ and relation $R = \{(2, 3)\}$ in A. Check whether relation R is reflexive, symmetric and transitive.

SECTION - C

4 × 3 = 12

13. Show that the relation S in the set R of real numbers, defined as $S = \{(a, b); a, b \in R \text{ and } a \leq b^3\}$ is neither reflexive, nor symmetric, nor transitive.
14. Prove that the relation R in R defined by $R = \{(a, b) : a \leq b^3\}$ is neither reflexive nor symmetric nor transitive.
15. Show that the function $f : R \rightarrow R$ given by $f(x) = x^3$ is injective.
16. If R is a relation defined on the set of natural numbers N as follows:
 $R = \{(x, y), x \in N, Y \in N \text{ and } 2x + y = 24\}$, then find the domain and range of the relation R. Also, find if R is an equivalence relation or not.

SECTION - D

 $2 \times 5 = 10$

17. Let $A = \{1, 2, 3, \dots, 9\}$ and R be the relation in $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$, for $(a, b), (c, d) \in A \times A$. Prove that R is an equivalence relation, also obtain the equivalent class $[(2, 5)]$.

OR

Let $A = \mathbb{R} - \{3\}$, $B = \mathbb{R} - \{1\}$. If $f: A \rightarrow B$ be defined by $f(x) = \frac{x-2}{x-3}, \forall x \in A$. Then, show that f is bijective.

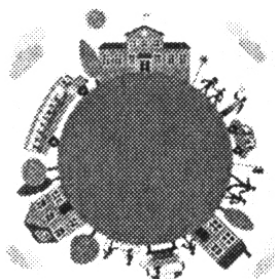
18. Show that each of the relation R in the set $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$ given by $R = \{(a, b); |a - b| \text{ is a multiple of } 4\}$ is an equivalence relation. Find the set of all elements related to 1.

SECTION - E

Case Study Based Questions

19. Read the following passage and answer the questions

Manikanta and Sharmila are studying in the same Kendriya Vidyalaya in Visakhapatnam. The distance from Manikanta's house to the school is same as distance from Sharmila's house to the school. If the houses are taken as a set of points and KV is taken as origin, then answer the below questions based on the given information; (M for Manikanta's house and S for Sharmila's house).



- i) Find a relation R is given by $R = \{(M, S) : \text{Distance of point M from origin is same as distance of point S from origin}\}$. 1
- ii) Let $R = \{(0, 3), (0, 0), (3, 0)\}$. Write a point which does not lie on the circle. 1
- iii) Find the maximum number of equivalence relations on the set $A = \{1, 2, 3\}$ 2